Cooperative Scheduling for Directional Wireless Charging with Spatial Occupation

Sixu Wu, Haipeng Dai, *Senior Member, IEEE,* Linfeng Liu, *Member, IEEE,* Lijie Xu, Fu Xiao, *Member, IEEE,* Jia Xu, *Senior Member, IEEE*

Abstract—Wireless Power Transfer (WPT) technology has been developed rapidly in recent years. The cooperative charging model and corresponding scheduling methods have been proposed to save the charging cost in paid charging service. However, the state-of-the-art methods ignore the spatial occupation issue of rechargeable devices. Moreover, the cooperative charging scheduling in directional wireless charging has not been studied yet. This paper studies the cooperative scheduling for directional wireless charging with spatial occupation. We formulate the Cooperative Charging Scheduling with Spatial occupation (CCSS) problem of Mobile Rechargeable Sensor Devices (MRSDs) for optimizing the total cost of whole charging system. We first investigate the properties of optimal arrangement of MRSDs in charging group and calculate the tight intervals of charging angles of MRSDs. We show that it is sufficient to bound the error by conducting angle discretization for only two MRSDs in each charging group. Then, a $(\ln n+1)(1+\varepsilon)$-approximation algorithm of the CCSS problem is proposed based on greedy approach, where n is the number of MRSDs, and ε is the discretization error. The results of extensive simulations and field experiments demonstrate that our algorithm can reduce at most 42.5% total cost comparing with the benchmark algorithms.

Index Terms—wireless rechargeable sensor network, cooperative charging, spatial occupation, directional wireless charging, angle discretization.

1 Introduction

Wireless Sensor Networks (WSNs) have been applied in various scenarios, such as military, agriculture, and transportation [1]. The energy supplement problem for a large number of sensor devices has raised widespread concern from both academic and industrial communities. Although some sensor devices can absorb various forms of energy from the surrounding environment, such as solar energy and wind energy [2], the energy extraction efficiency is largely affected by the environment and weather, and is highly unpredictable and unstable. Wireless Power Transmission (WPT) can provide continuous and reliable power supply for WSNs. With the advance of WPT technology, Wireless Rechargeable Sensor Networks (WRSNs) have been extensively developed in real life, such as unmanned aerial vehicles [3], driverless Electric Vehicles (EVs) [4], industrial robots [5], RFID systems [6], and building structure monitoring [7].

With the booming Internet of Things (IoT) and WPT, the number of Mobile Rechargeable Sensor Devices (MRSDs), including inspection robots and unmanned vehicles, increases sharply in recent years. In future, the wireless charger will become the infrastructure to provide paid charging service for rechargeable devices. The Radio Frequency (RF) charging technology [8] can provide the energy supply for multiple MRSDs in the near open field simultaneously without additional discharging cost [1]. Therefore, multiple MRSDs in the common charging hours can share the charging cost, reducing the individual cost. The cooperative charging can largely improve the energy utilization and reduce the actual charging expenditure by exploring the characteristics of RF charging technology and designing the cooperative pricing structures. Such cooperative charging is a natural and economical service model for the RF charging technology. The cooperative charging is important, particularly in commercial wireless charging, since it increases the market competitiveness of Charging Service Providers (CSPs). How to model the commercial RF charging service and optimize the charging cost is essential to popularizing WPT technology further.

Moreover, the rechargeable sensor devices (including sensing unit, processing unit, communication unit, battery and power receiving antenna) need to take up certain space. Comparing with the limited charging distance (usually several meters), the spatial occupation of rechargeable sensor devices cannot be omitted. Due to the spatial occupation, the number of MRSDs assigned to a single charger is limited. On the other hand, the rechargeable sensor devices with different charging angles have different charging power in directional wireless charging. The key problem of cooperative scheduling in directional wireless charging with spatial occupation is how to arrange the MRSDs assigned to the charger to reduce the charging cost, which has not been studied yet.

This paper aims to study the cooperative scheduling of MRSDs in directional wireless charging with spatial occupation under the commercial RF charging service model.

- *S. Wu, L. Liu, L. Xu, F. Xiao and J. Xu are with the Jiangsu Key Laboratory of Big Data Security and Intelligent Processing, Nanjing University of Posts and Telecommunications, Nanjing, Jiangsu 210023, China. (E-mail: 2021070711@njupt.edu.cn, liulf@njupt.edu.cn, ljxu@njupt.edu.cn, xiaof@njupt.edu.cn, xujia@njupt.edu.cn).*
- *H. Dai is with the State Key Laboratory for Novel Software Technology, Nanjing University, Nanjing, Jiangsu 210023 China. (E-mail: haipengdai@nju.edu.cn).*
- *Corresponding author: Jia Xu*

Manuscript received August 23, 2022; revised .

Some studies have focused on the problem of scheduling for mobile rechargeable devices [3] [9]. However, most of them aim to optimize the charging efficiency or charging time rather than the charging cost from the perspective of economics through cooperative charging. To save the charging cost, the cooperative charging scheduling [10] has been proposed recently. However, it does not consider either the spatial occupation issue of rechargeable sensor devices or the cooperative charging scheduling in directional wireless charging. In addition, different from the traditional wireless charging systems, commercial wireless charging system provides the paid wireless charging service with specific pricing rule. The application scenarios of commercial wireless charging systems include wireless charging for electric vehicles and sensor networks [11], [12]. However, in the commercial wireless charging environment, the payment in existing price rules simply depends on the charging time or replenished energy independently. In other words, the existing pricing mechanisms cannot promote the cooperation among rechargeable devices through saving charging expenditure.

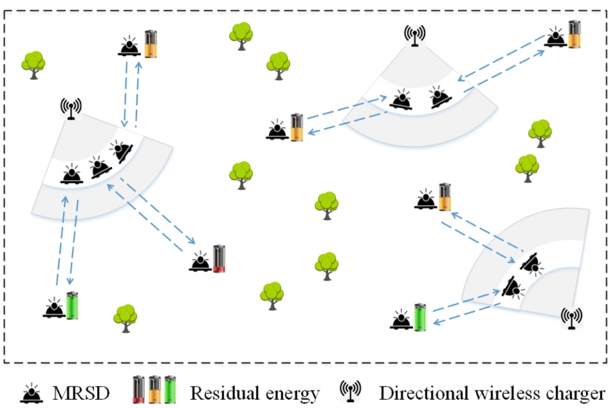

Fig. 1. Illustration of cooperative charging system

In this paper, we present a cooperative charging system shown in Fig. 1. We consider a set of directional wireless chargers located at fixed positions in a 2D plane. These chargers follow identical pricing structure. The chargers are operated by different CSPs and may have different charging price [10]. The MRSDs can move from the initial positions to the charging facilities of chargers to obtain the charging service. The MRSDs assigned to the same charger form a charging group, in which the MRSDs can obtain the surplus by sharing the charging cost in the common charging hours [10]. We consider that each MRSD takes up some space, thus, the size of charging group is limited. When the energy is replenished, the MRSDs will come back to the initial positions to continue the sensing tasks. The objective is to find an assignment of MRSDs and the spatial conflict-free arrangements of charging group to minimize the charging cost (payment to the chargers) such that all MRSDs' energy demands can be satisfied.

The problem of cooperative scheduling for directional wireless charging with spatial occupation is very challenging. First, we need to assign the MRSDs to multiple charging groups. This problem is more difficult than the Capacitated Facility Location Problem (CFLP) [13], which is a well-known NP-hard problem. Second, in directional charging, different charging angles of MRSD result in different charging power, charging time, and charging cost. To optimize the cost, we need to not only assign the MRSDs to the chargers but also decide the charging angles of assigned MRSDs. However, the searching space of charging angles is continuous, therefore, the possible charging angles are infinite. Moreover, due to the spatial occupation of MRSDs, we need to guarantee that the arrangement of MRSDs is conflict-free. Therefore, a precise angle search interval of each MRSD is needed to avoid the unfeasible solutions.

The main contributions of this paper are as follows:

- To the best of our knowledge, this is the first work to study the cooperative scheduling for directional wireless charging with spatial occupation. This work will benefit the wireless charging economy through promoting the economic cooperation of MRSDs and reducing the charging cost for the commercial wireless charging services.
- We model the Cooperative Charging system for directional wireless charging and formalize the Cooperative Charging Scheduling with Spatial occupation (CCSS) problem. We show that the CCSS problem is NP-hard.
- We explore the properties of optimal arrangement and find the spatial conflict-free arrangement of MRSDs. Through calculating the precise angle search interval of each MRSD, the efficiency of angle discretization of charging group can be largely improved without loss. Our conclusions are generic and can be commonly used for the problems related to the directional wireless charging with spatial occupation, such as minimizing charging delay or maximizing charging utility.
- We propose the greedy approach-based Charging Scheduling Algorithm (CSA) through the angle discretization, which discretizes the infinite charging angles of MRSDs with bounded discretization error. We show that CSA is $(\ln n + 1)(1 + \varepsilon)$-approximation for the CCSS problem.
- Through extensive simulations and field experiments, we demonstrate that the proposed algorithm can reduce at most 42.5% total cost comparing with the benchmark algorithms.

The rest of the paper is organized as follows. Section 2 presents the brief review on the state-of-art research. Section 3 presents the system model and formulates the CCSS problem. Section 4 presents the details of our solution. The simulation results are presented in Section 5. Field experiments are shown in Section 6. We discuss the solution for omnidirectional wireless charging case in Section 7. We conclude this paper in Section 8.

2 Related Work

There has been extensive research on wireless charging for scheduling the placement [14], charging power [9] or switches [15] of chargers, or trajectory of mobile chargers [16], [17] or mobile rechargeable devices [3], [9], [18], with optimization objective of charging cost [16], [19], [20], [21], charging time [8], [22], or charging utility [23], [24], etc. The existing research can be classified as paid charging [11], [12] and non-paid charging [10], [16]. From the perspective of cooperation, there has been also cooperative charging [10] and non-cooperative charging [14], [21]. According to the charging model, it can be classified as directional charging [8], [21], [25] and omnidirectional charging [10], [15]. We briefly review the works on charging cost optimization, scheduling for mobile rechargeable devices, paid charging, cooperative charging, and directional charging, which are closely related to this study.

Charging cost optimization. Wang *et al.* [16] studied the energy minimization problem of wireless charging in a dense WSN. They first found the initial charging clusters and the charging path, and then improved the path to reduce energy consumption. However, the cost defined in this work is the proportion of energy consumption rather than the actual charging expenditure. Zhou *et al.* [19] relaxed the strictness of perpetual operation by allowing some sensor nodes to temporarily run out of energy while still maintaining target k-coverage in the network at lower cost of Mobile Charger (MC). Their extensive simulation results demonstrated significant improvements of network scalability and cost saving. However, in our paper, the energy demands of all MRSDs should be satisfied. Jia *et al.* [20] concerned the fundamental issue of charging path design with the minimized energy cost, i.e., given a set of rechargeable sensors, they appropriately designed the MC's charging path to minimize the energy cost which was due to the wireless charging and the MC's movement, such that the different charging demand of each sensor was satisfied. This paper also regarded the cost as the proportion of energy consumption. In [21], the authors studied the minimal charging expenditure problem with directional chargers to decrease the energy expenditure of the charger such that the charging demands of all sensors are satisfied. However, the proposed scheduling is for static sensors and cannot be applied to mobile rechargeable devices.

Scheduling for mobile rechargeable devices. Jin *et al.* [3] proposed the scheduling scheme of Unmanned Aerial Vehicles (UAVs), which can fly to the buses to replenish energy. In [9], the authors studied the issue of Charging on the Move (CM) to optimize the scheduling of transmitting power of static chargers for mobile devices with given movement trajectories. In [18], the strategies of selecting the local-optimal EVs with surplus energy for the EVs with insufficient energy and rescheduling their travel routes were investigated, and a distributed Reciprocal Charging Mechanism (RCM) was proposed. However, in our paper, the MRSDs cannot charge each other. Moreover, these studies did not consider the commercial paradigm of wireless charging from the cooperative perspective.

Paid charging. Fan *et al.* [11] proposed a dynamic pricing mechanism to maximize the long-term profit of the charging platform by jointly controlling the demand queues of multiple charging stations. Gupta *et al.* [12] proposed a pricing model for dedicated charging of rechargeable sensor devices and used game theory approach to find the Nash equilibrium price as well as the individual profit for each charger. However, all of works aforementioned did not study the issue of cooperative scheduling for paid charging. Moreover, most of existing pricing mechanisms cannot provide the surplus from the cooperation.

Cooperative charging. Recently, Xu *et al.* [10] presented a wireless charging service model and proposed the algorithm for joint optimization of rechargeable devices' charging cost and moving cost. However, they only studied the cooperative charging with omnidirectional chargers and did not consider the spatial occupation issue as well.

Directional charging. Since the possible charging angles of rechargeable devices in directional charging are continuous and infinite, the angle discretization is widely used to solve the directional charging scheduling problems [8]. However, the existing angle discretization method cannot be applied straightforwardly to solve our CCSS problem. Due to the spatial occupation of MRSDs, we should arrange multiple MRSDs simultaneously and make all MRSDs satisfy the discretization error.

Overall, there is no rechargeable device scheduling in the literature for the directional wireless charging with spatial occupation.

3 System Model and Problem Formulation

3.1 System Model

TABLE 1
Frequently Used Notations

Symbol	Description
M, m	Set of chargers, number of chargers
N, n	Set of MRSDs, number of MRSDs
E_i	Energy demand of MRSD o_i
E_i^{re}	Residual energy of MRSD o_i
E_{MAX}	Energy capacity of MRSDs
b_i	Unit moving energy consumption of MRSD o_i
$\|\|s_j o_i\|\|$	Distance between charger s_j and MRSD o_i
d_j	Charging distance of charger s_j
α, β, μ	Charging parameters of chargers
θ_{ij}	Charging angle between charger s_j and MRSD o_i
$\theta_j^{\triangle}$	Minimum angle interval of MRSDs in charging group G_j
δ	Maximum charging angle of chargers
$Pr(s_j, o_i, \theta_{ij})$	Charging power from charger s_j to MSRD o_i with charging angle θ_{ij}
G_j	Charging group of charger s_j
a_j	Unit continued charging price of charger s_j
A_j	Base fare of charger s_j
T_j	Charging time threshold of charger s_j
ϕ_j	Charging angle profile (arrangement) of charging group G_j
$T_i(s_j, \theta_{ij})$	Charging time of MSRD o_i in charging group G_j with charging angle θ_{ij}
$T(G_j, \phi_j)$	Charging time of the charging group G_j
$c(G_j, \phi_j)$	Cost of charging group G_j
$E_i(s_j)$	Actual replenished energy from charger s_j required by MRSD o_i
ε	Discretization error

We consider a set $M = \{s_1, s_2, ..., s_m\}$ of m directional wireless chargers located at fixed positions in a 2D plane Ω. These chargers are operated by different Charging Service Providers (CSPs), and therefore, they may have different charging prices. Suppose that there are a set $N = \{o_1, o_2, ..., o_n\}$ of n Mobile Rechargeable Sensor Devices (MRSDs) located in the same 2D plane. Each MRSD

$o_i \in N$ has the energy demand E_i, the residual energy E_i^{re}, and the unit moving energy consumption b_i. Note that the energy demand E_i is not the actual replenished energy from the charger, but the increased energy of o_i after it comes back to the initial position comparing to the residual energy E_i^{re}. We consider that the MRSDs are homogeneous and have the same energy capacity E_{MAX}.

As shown in [26], if the RF antenna of MRSD is blocked by obstacles, the received power will be largely reduced. Since the MRSDs have non-negligible spatial occupation, each charger s_j has a charging facility to fix the MRSDs with charging distance d_j to avoid the block between MRSDs. The MRSDs need to move to the charging facilities for energy replenishment. Actuality, many mobile rechargable devices with non-negligible spatial occupation have such charging facilities, such as the wireless charging parking place for Electric Vehicles (EVs) [27], wireless charging pad for Unmanned Aerial Vehicles (UAVs) [28], wireless charging area for rechargeable robots [29].

In most cases, the mobile sensor networks are deployed in large-scale scenarios, such as disaster assessment [30], life detection [31], battlefield situational awareness [32], and inspection robot for large substation [33] or warehouse [34]. Therefore, the charging distance d_j can be omitted when we calculate the moving distance of MRSDs. Specifically, the moving distance from any MRSD o_i to any charger s_j is defined as the Euclidean distance between them, denoted by $||s_j o_i||$.

According to the directional charging model given in [8], the charging power from any charger s_j to any MSRD o_i is

$$Pr(s_j, o_i, \theta_{ij}) = \begin{cases} \dfrac{\mu(cos\theta_{ij} + \alpha)}{(\beta + d_j)^2}, & d_j \le D, |\theta_{ij}| \le \delta \\ 0, & otherwise, \end{cases} \tag{1}$$

where α, β, and μ are three parameters determined by the magnetic environment and hardware. θ_{ij} is the charging angle between s_j and o_i, and $\delta \in (0, \frac{\pi}{2}]$ is the maximum charging angle of chargers. D is the maximum charging distance. Since d_j is a known constant and is not more than D definitely in our system model, the MRSDs can always obtain positive power if it is within the maximum charging angle of chargers.

To keep working (e.g., perform sensing tasks), the MRSDs must return to the initial locations after the energy is replenished. Let G_j be the charging group of s_j, i.e., the set of MRSDs charged by charger s_j. Then the actual replenished energy from charger s_j required by o_i is the sum of requested energy and the round-trip moving energy consumption between the initial location and the charging position, which can be calculated by

$$E_i(s_j) = E_i + 2b_i||s_j o_i||. \tag{2}$$

The charging time of o_i in charging group G_j with charging angle θ_{ij} is

$$T_i(s_j, \theta_{ij}) = \frac{E_i(s_j)}{Pr(s_j, o_i, \theta_{ij})}. \tag{3}$$

We use ϕ_j to denote the charging angle profile (also called arrangement) of MRSDs in G_j. For a charging group G_j and a charging angle profile ϕ_j, the charging time of the charging group is the maximum charging time of all MRSDs in the charging group:

$$T(G_j, \phi_j) = \max_{o_i \in G_j} T_i(s_j, \theta_{ij}). \tag{4}$$

Although the arriving time of MRSDs may be different, according to [10], the moving time of MRSDs can be ignored because it is very small compared to the charging time.

To promote cooperation between the MRSDs, we present a charging time-based pricing rule of chargers, where the cost of the charging group only depends on the maximum charging time of MRSDs. Moreover, since the size of charging group is limited due to the spatial occupation of MRSDs, we introduce the base fare pricing structure to ensure the revenue of CSPs. The base fare pricing structure has been widely used in many fields, such as taxi pricing [35] and express delivery industry [36]. The cost of charging group G_j with arrangement ϕ_j is defined as

$$c(G_j, \phi_j) = \begin{cases} A_j, & T(G_j, \phi_j) \le T_j \\ A_j + a_j(T(G_j, \phi_j) - T_j), & otherwise, \end{cases} \tag{5}$$

where A_j and a_j are the base fare and unit continued charging price of s_j, respectively. T_j is the charging time threshold of s_j. These parameters are determined by the CSPs based on the charging market, and the detail is not of academic interest. Following the realistic base fare models, there is

$$A_j \ge a_j T_j. \tag{6}$$

Considering the spatial occupation of MRSDs, we need to calculate the minimum angle interval between any two adjacent MRSDs to avoid the space occupation conflict. Without loss of generality, as shown in Fig. 2, we consider that each MRSD (including the antenna) occupies the space with length x and the antenna is located in the center of the front of MRSD. Therefore, the minimum angle interval of MRSDs in charging group G_j can be calculated as

$$\theta_j^{\triangle} = 2tan^{-1}\frac{x}{2d_j}. \tag{7}$$

Obviously, if the angle difference of any two adjacent MRSDs is not less than the minimum angle interval, the arrangement must be conflict-free. An illustration of charging group G_j with three MRSDs is shown in Fig. 3.

We list the frequently used notations in Table 1.

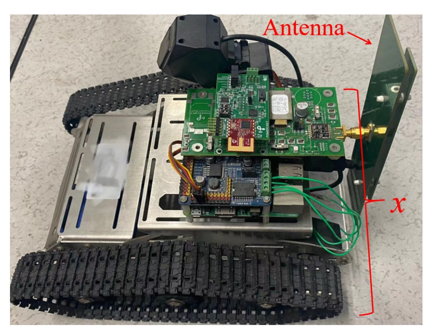

Fig. 2. Illustration of MRSD with spatial occupation

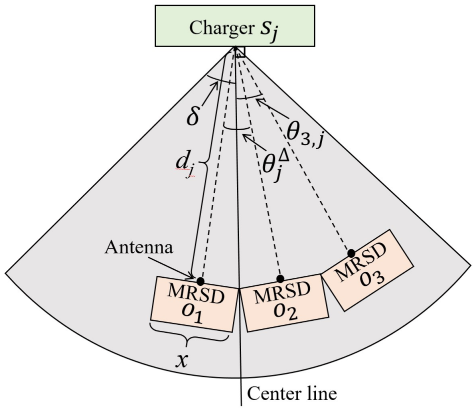

Fig. 3. Illustration of charging group G_j

3.2 Problem Formulation

The problem is to find an assignment of MRSDs and the spatial conflict-free arrangements of charging group to minimize the total cost of all charging groups such that each MRSD is assigned to exactly one charger without conflict of spatial occupation. We refer to this problem as Cooperative Charging Scheduling with Spatial occupation (CCSS) problem, which can be formulated as

$$(CCSS): \quad \min \sum_{s_j \in M} c(G_j, \phi_j), \tag{8}$$

$$s.t. \quad \bigcup_{s_j \in M} G_j = N, \tag{8-1}$$

$$G_j \cap G_{j'} = \emptyset, \ \forall s_j \neq s_{j'}, \ s_j \in M, \ s_{j'} \in M, \tag{8-2}$$

$$|\theta_{ij} - \theta_{i'j}| \geq \theta_j^{\triangle}, \ \forall o_i \neq o_{i'}, \ o_i, o_{i'} \in G_j, s_j \in M, \tag{8-3}$$

$$|\theta_{ij}| \leq \delta, \ \forall o_i \in G_j, \ s_j \in M. \tag{8-4}$$

The constraint (8-1) ensures that all MRSDs should be charged. The constraint (8-2) ensures that each MRSD can be scheduled to exact one charger. In view of the commercial feasibility of charging economy, we consider that each MRSD can only obtain the charging service from one charger every time. The constraint (8-3) ensures that the arrangement is conflict-free. The constraint (8-4) ensures that the MRSDs are within the maximum charging angles of chargers.

To guarantee the existence of feasible solutions of CCSS problem, three conditions should be satisfied. First, the residual energy of any MRSD should be sufficient to reach the furthest charger, i.e.,

$$E_i^{re} \geq \max_{s_j \in M} b_i ||s_j o_i||, \forall o_i \in N. \tag{9}$$

Second, the energy demand of any MRSD can be satisfied after returning the initial position, i.e.,

$$E_i \leq E_{\text{MAX}} - \max_{s_j \in M} b_i ||s_j o_i|| - E_i^{re}, \forall o_i \in N. \tag{10}$$

Third, there is enough space to place all MRSDs, i.e.,

$$\sum_{s_j \in M} \lfloor \frac{2\delta}{\theta_j^{\triangle}} + 1 \rfloor \geq n. \tag{11}$$

If the above three conditions cannot be satisfied, we can conduct a simple preprocessing before solving the problem: Remove the MRSDs that do not satisfy (9) or (10). Afterwards, if (11) is not satisfied, remove the MRSDs with most residual energy until there is enough space.

4 Solution of CCSS Problem

In this section, we present the approximation algorithm of the CCSS problem. We first show the hardness of CCSS problem and introduce the design rationale of solution. Then, we explore the properties of optimal arrangement of MRSDs in charging group and propose the angle discretization method to obtain the finite candidate angles and the corresponding approximate charging power. Finally, we give the details of algorithm design and analysis.

4.1 Hardness

First, we attempt to find an optimal algorithm for the CCSS problem. Unfortunately, as the following theorem shows, the CCSS problem is NP-hard.

Theorem 1. *The CCSS problem is NP-hard.*

Proof: We first introduce the following Capacitated Generalized Facility Location Problem (CGFLP): There are a set M of facilities and a set N of clients. The connection cost of any client $o_i \in N$ to any facility $s_j \in M$ is $\frac{\max\{0, a_j(T(G_j,\phi_j) - T_j)\}}{\sum_{o_i \in N} x_{ij}}$. The facility cost of facility s_j is A_j. The capacity of facility s_j is $\lfloor \frac{2\delta}{\theta_j^{\triangle}} + 1 \rfloor$. The objective is to find an assignment of each client to an open facility to minimize the total cost incurred. The CGFLP can be formulated as follows

$$(CGFLP): \min \sum_{s_j \in M} \sum_{o_i \in N} \frac{\max\{0, a_j(T(G_j,\phi_j) - T_j)\}}{\sum_{o_i \in N} x_{ij}} x_{ij} + \sum_{s_j \in M} A_j y_j, \tag{12}$$

$$\sum_{o_i \in N} x_{ij} \leq \lfloor \frac{2\delta}{\theta_j^{\triangle}} + 1 \rfloor, \forall s_j \in M, \tag{12-1}$$

$$s.t. \quad \sum_{s_j \in M} x_{ij} = 1, \forall o_i \in N, \tag{12-2}$$

$$x_{ij} \leq y_j, \forall s_j \in M, \forall o_i \in N, \tag{12-3}$$

$$x_{ij} \in \{0,1\}, \forall s_j \in M, \forall o_i \in N, \tag{12-4}$$

$$y_j \in \{0,1\}, \forall s_j \in M, \tag{12-5}$$

where y_j is a binary variate indicating whether facility s_j is open. x_{ij} is a binary variate indicating whether client o_i is assigned to facility s_j.

Based on (5), the objective function of CCSS problem can be represented as follows

$$\sum_{s_j \in M} c(G_j, \phi_j) = \sum_{s_j \in M} \sum_{o_i \in N} \frac{\max\{0, a_j(T(G_j,\phi_j) - T_j)\}}{\sum_{o_i \in N} x_{ij}} x_{ij} + \sum_{s_j \in M} A_j y_j. \tag{13}$$

Thus, the CCSS problem defined in (8) is equivalent to the CGFLP. If the connection cost of any client $o_i \in N$ to any facility $s_j \in M$ is a constant, the problem defined in (12) is simplified to the Capacitated Facility Location Problem (CFLP) [13]. In CCSS problem, $\frac{\max\{0, a_j(T(G_j, \phi_j) - T_j)\}}{\sum_{o_i \in N} x_{ij}}$ is related to the clients assigned to the facility s_j and cannot be known in advance. Since the CFLP is NP-hard, the CCSS problem is NP-hard. ■

4.2 Design Rationale

Since the CCSS problem is NP-hard, it is impossible to compute the optimal solution in polynomial time. We turn our attention to the approximation algorithm design.

Our solution follows the greedy approach. We iteratively assign a MRSD set to a charger to minimize the ratio of the marginal cost to the number of newly covered MRSDs (termed cost effectiveness) until all MRSDs are assigned.

Based on (11), the number of MRSDs that can be assigned to any charger s_j is at most $\lfloor 2\delta/\theta_j^{\triangle} \rfloor + 1$. On the other hand, based on (5), the cost of charging group is monotone nondecreasing with the charging time of charging group. Further, given the size of charging group, the charging time of charging group consisting of the MRSDs with low actual replenished energy is obvious smaller than the charging group consisting of MRSDs with high actual replenished energy. Therefore, given the number of MRSDs $k \in \{1, 2, ..., \lfloor 2\delta/\theta_j^{\triangle} \rfloor + 1\}$, we can assign the top k MRSDs with lowest actual replenished energy to the charger s_j, achieving the lowest marginal cost.

Afterwards, given the set of k MRSDs, we need to decide the charging angles of these MRSDs to minimize the charging time of the charging group. However, the searching space of charging angles is continuous. To solve this problem, we use the angle discretization technique to get the finite candidate charging angles and the corresponding approximate charging power.

4.3 Properties of Optimal Arrangement

We first explore the properties of optimal arrangement of MRSDs in charging group, where the optimal arrangement is the arrangement with the minimum charging time of charging group.

Lemma 1. *For any two adjacent MRSDs $o_i, o_{i'} \in G_j$, the angle interval of o_i and $o_{i'}$ in the optimal arrangement of charging group G_j is $\theta_j^{\triangle}$, i.e., $|\theta_{ij} - \theta_{i'j}| = \theta_j^{\triangle}$.*

Proof: We first show $|\theta_{ij} - \theta_{i'j}| < \theta_j^{\triangle}$ is unfeasible because such arrangement will result in spatial occupation conflict.

Next, we show $|\theta_{ij} - \theta_{i'j}| > \theta_j^{\triangle}$ is unreasonable. Supposing $|\theta_{ij} - \theta_{i'j}| > \theta_j^{\triangle}$ in the optimal arrangement for any two adjacent MRSDs $o_i, o_{i'} \in G_j$, we consider the following two cases:

Case 1: o_i and $o_{i'}$ are placed on the different sides of center line of charger s_j.

In this case, we can rotate all MRSDs towards to the center line of charger, and make o_i and $o_{i'}$ encounter right at the center line. By this way, the charging power of all MRSDs in G_j will increase, and the charging time of all MRSDs in charging group G_j will reduce accordingly based on (3). Further, the charging time of charging group G_j will reduce based on (4).

Case 2: o_i and $o_{i'}$ are placed on the same side of center line of charger s_j.

In this case, we can rotate the MRSDs farther from center line of charger towards to the center line until the first MRSD encounters another MRSD. By this way, the charging power of all rotated MRSDs in G_j will increase. Therefore, the charging time of charging group G_j may reduce. ■

Lemma 2. *For any two MRSDs $o_i, o_{i'} \in G_j$, $E_i(s_j) \leq E_{i'}(s_j)$, θ_{ij} and $\theta_{i'j}$ are the charging angles of o_i and $o_{i'}$ in the optimal arrangement, respectively, then $|\theta_{ij}| \geq |\theta_{i'j}|$ (i.e., $Pr(s_j, o_i, \theta_{ij}) \leq Pr(s_j, o_{i'}, \theta_{i'j})$).*

Proof: We assume $|\theta_{ij}| < |\theta_{i'j}|$ in the optimal arrangement for any two MRSDs $o_i, o_{i'} \in G_j$, $E_i(s_j) \leq E_{i'}(s_j)$, and denote the corresponding charging time of G_j by $T(G_j, \phi_j)$. In this case, the maximum charging time of o_i and $o_{i'}$ is $\max\{\frac{E_i(s_j)}{Pr(s_j, o_i, \theta_{ij})}, \frac{E_{i'}(s_j)}{Pr(s_j, o_{i'}, \theta_{i'j})}\} = \frac{E_{i'}(s_j)}{Pr(s_j, o_{i'}, \theta_{i'j})}$. We exchange the charging angles of o_i and $o_{i'}$, and denote the charging time of G_j by $T(G_j, \phi'_j)$. In this case, the maximum charging time of o_i and $o_{i'}$ is $\max\{\frac{E_i(s_j)}{Pr(s_j, o_{i'}, \theta_{i'j})}, \frac{E_{i'}(s_j)}{Pr(s_j, o_i, \theta_{ij})}\}$. Since $\frac{E_i(s_j)}{Pr(s_j, o_{i'}, \theta_{i'j})} \leq \frac{E_{i'}(s_j)}{Pr(s_j, o_{i'}, \theta_{i'j})}$ and $\frac{E_{i'}(s_j)}{Pr(s_j, o_i, \theta_{ij})} < \frac{E_{i'}(s_j)}{Pr(s_j, o_{i'}, \theta_{i'j})}$, and the charging time of other MRSDs in G_j is unchanged, we have $T(G_j, \phi'_j) < T(G_j, \phi_j)$, which contradicts the definition of optimal arrangement. ■

Based on Lemma 1 and Lemma 2, we can conclude that the MRSDs in any charging group are tightly arranged and the MRSD with larger actual replenished energy is arranged closer to the center line of the charger in the optimal arrangement. For convenience, we only discuss the situation where the MRSD with largest actual replenished energy is always placed on the anticlockwise side of charger's center line in this paper since the arrangement in the charging group is symmetrical. Without loss of generality, as illustrated in Fig. 4, we consider that there are k MRSDs in charging group $G_j = \{o_1, o_2, ..., o_k\}$ with nonincreasing order of actual replenished energy. Then, $o_2, o_3, ..., o_k$ will be arranged tightly and alternately on two sides of o_1 in order. Therefore, we have the following lemma.

Lemma 3. *Given k MRSDs in charging group $G_j = \{o_1, o_2, ..., o_k\}$, $E_1(s_j) \geq E_2(s_j) \geq ... \geq E_k(s_j)$, $k \geq 1$, for any $o_i \in G_j$, there is $\theta_{i,j} = \theta_{1,j} + (-1)^{i+1} \lfloor \frac{i}{2} \rfloor \theta_j^{\triangle}$ in the optimal arrangement.*

4.4 Angle Discretization

Based on the previous analysis, we only need to decide the charging angle of the MRSD with largest actual replenished energy (denoted by o_1 for convenience), and the other MRSDs' charging angle can be determined based on Lemma 3. However, the possible charging angles are continuous and infinite. To solve this problem, we use the angle discretiza-

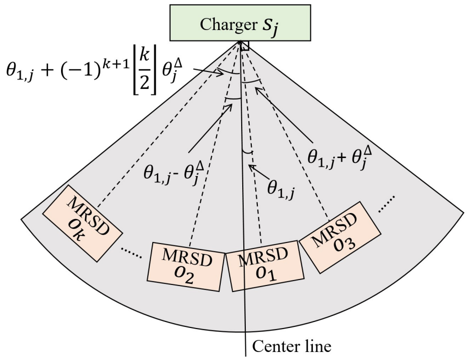

Fig. 4. Illustration of charging angles of MRSDs in charging group G_j, where $E_1(s_j) \geq E_2(s_j) \geq ... \geq E_k(s_j)$

tion technique to obtain the finite candidate angles and the corresponding approximate charging power.

First of all, to guarantee that all MRSDs in the charging group are in the maximum charging angle of charger, the charging angle of o_1 should be constrained in a specific interval. We have the following lemma.

Lemma 4. *Given the charging group* $G_j = \{o_1, o_2, ..., o_k\}$, $E_1(s_j) \geq E_2(s_j) \geq ... \geq E_k(s_j)$, $k \geq 1$, *there must be* $\theta_{1,j} \in [\lfloor\frac{k-1}{2}\rfloor\theta_j^{\triangle} - \delta, \delta - \lfloor\frac{k}{2}\rfloor\theta_j^{\triangle}]$ *if k is odd, and* $\theta_{1,j} \in [\lfloor\frac{k}{2}\rfloor\theta_j^{\triangle} - \delta, \delta - \lfloor\frac{k-1}{2}\rfloor\theta_j^{\triangle}]$ *otherwise.*

Proof: If $k = 1$, the optimal arrangement is $\theta_{1,j} = 0$, and we obtain the lemma straightforwardly.

If $k > 1$, we only need to guarantee that $|\theta_{k,j}| \leq \delta$ and $|\theta_{k-1,j}| \leq \delta$ based on Lemma 3, i.e.,

$$|\theta_{k,j}| = |\theta_{1,j} + (-1)^{k+1}\lfloor\frac{k}{2}\rfloor\theta_j^{\triangle}| \leq \delta. \tag{14}$$

$$|\theta_{k-1,j}| = |\theta_{1,j} + (-1)^{k}\lfloor\frac{k-1}{2}\rfloor\theta_j^{\triangle}| \leq \delta. \tag{15}$$

We consider the following two cases.

Case 1: k is odd.

o_k is on the anticlockwise side of charger's center line, and $\theta_{1,j} + (-1)^{k+1}\lfloor\frac{k}{2}\rfloor\theta_j^{\triangle} > 0$. Further, o_{k-1} is on the clockwise side of charger's center line, and $\theta_{1,j} + (-1)^{k}\lfloor\frac{k-1}{2}\rfloor\theta_j^{\triangle} < 0$.

Case 2: k is even.

o_k is on the clockwise side of charger's center line, and $\theta_{1,j} + (-1)^{k+1}\lfloor\frac{k}{2}\rfloor\theta_j^{\triangle} < 0$. Further, o_{k-1} is on the anticlockwise side of charger's center line, and $\theta_{1,j} + (-1)^{k}\lfloor\frac{k-1}{2}\rfloor\theta_j^{\triangle} > 0$.

Solving (14) and (15) for the aforementioned two cases, respectively, we obtain the lemma. ■

Theoretically, we can discretize the intervals given in Lemma 4 to obtain the approximate solution of $\theta_{1,j}$. However, the intervals given in Lemma 4 are not tight. To reduce the searching space of $\theta_{1,j}$, we further tighten range of $\theta_{1,j}$.

Lemma 5. *Given the charging group* $G_j = \{o_1, o_2, ..., o_k\}$, $E_1(s_j) \geq E_2(s_j) \geq ... \geq E_k(s_j)$, $k \geq 1$, *there must be* $\theta_{1,j} \in [0, \min\{\frac{\theta_j^{\triangle}}{2}, \delta - \lfloor\frac{k}{2}\rfloor\theta_j^{\triangle}\}]$, *if k is odd, and* $\theta_{1,j} \in [\max\{0, \lfloor\frac{k}{2}\rfloor\theta_j^{\triangle} - \delta\}, \min\{\frac{\theta_j^{\triangle}}{2}, \delta - \lfloor\frac{k-1}{2}\rfloor\theta_j^{\triangle}\}]$ *otherwise.*

Proof: We first show $\theta_{1,j} \in [0, \frac{\theta_j^{\triangle}}{2}]$ in the optimal arrangement. Since we only study the situation where the MRSD with largest actual replenished energy is always placed on the anticlockwise side of charger's center line in this paper, we have $\theta_{1,j} \geq 0$. Next, we prove $\theta_{1,j} \leq \frac{\theta_j^{\triangle}}{2}$. Suppose $\theta_{1,j} > \frac{\theta_j^{\triangle}}{2}$. If $k = 1$, there must be $\theta_{1,j} = 0$ in the optimal arrangement, which contradicts the assumption. If $k > 1$, we have $\theta_{2,j} = \theta_{1,j} - \theta_j^{\triangle}$ based on Lemma 3 and $|\theta_{2,j}| < |\theta_{1,j}|$. Since $E_1(s_j) \geq E_2(s_j)$, this contradicts Lemma 2. Thus, $\theta_{1,j} \in [0, \frac{\theta_j^{\triangle}}{2}]$.

On the other hand, based on Lemma 4, we have $\theta_{1,j} \in [\lfloor\frac{k-1}{2}\rfloor\theta_j^{\triangle} - \delta, \delta - \lfloor\frac{k}{2}\rfloor\theta_j^{\triangle}]$ if k is odd, and $\theta_{1,j} \in [\lfloor\frac{k}{2}\rfloor\theta_j^{\triangle} - \delta, \delta - \lfloor\frac{k-1}{2}\rfloor\theta_j^{\triangle}]$ otherwise. Since $k \leq \lfloor 2\delta/\theta_j^{\triangle}\rfloor + 1$, we have the following inequations:

$$\lfloor\frac{k}{2}\rfloor\theta_j^{\triangle} - \delta \leq \lfloor\frac{\lfloor 2\delta/\theta_j^{\triangle}\rfloor + 1}{2}\rfloor\theta_j^{\triangle} - \delta \leq \frac{\theta_j^{\triangle}}{2}. \tag{16}$$

$$\delta - \lfloor\frac{k}{2}\rfloor\theta_j^{\triangle} \geq -\frac{\theta_j^{\triangle}}{2}. \tag{17}$$

$$\lfloor\frac{k-1}{2}\rfloor\theta_j^{\triangle} - \delta \leq 0. \tag{18}$$

$$\delta - \lfloor\frac{k-1}{2}\rfloor\theta_j^{\triangle} \geq 0. \tag{19}$$

Combining $\theta_{1,j} \in [0, \frac{\theta_j^{\triangle}}{2}]$ and Lemma 4 using (16), (17), (18) and (19), we obtain the Lemma. ■

For any MRSD $o_i \in G_j$, we use the angle discretization method in [8] to obtain the finite candidate set of charging angles. We denote the maximum charging power and minimum charging power of o_i in the discretization interval (e.g., the discretization interval of o_1 is given in Lemma 5) by Pr_{ij}^{MAX} and Pr_{ij}^{MIN}, respectively. We further denote the v-th discrete charging angle of o_i as θ_{ij}^{v} and the corresponding charging power as Pr_{ij}^{v}. Let n_i be the number of segments of o_i after angle discretization. Given the discretization error $\varepsilon > 0$, let

$$Pr_{ij}^{v} = Pr_{ij}^{\mathrm{MAX}}(1+\varepsilon)^{-v}, v = 1, 2, ..., n_i. \tag{20}$$

Specifically, we have $Pr_{ij}^{0} = Pr_{ij}^{\mathrm{MAX}}$, $Pr_{ij}^{n_i} = Pr_{ij}^{\mathrm{MIN}}$, and $Pr_{ij}^{v} = Pr_{ij}^{v-1}(1+\varepsilon)^{-1}$ for $v = 1, 2, ..., n_i - 1$.

According to (1), θ_{ij}^{v} can be calculated by

$$\theta_{ij}^{v} = cos^{-1}(\frac{Pr_{ij}^{v}(\beta + d_j)^2}{\mu} - \alpha). \tag{21}$$

Based on (20), we have

$$n_i = \lceil\frac{\ln(Pr_{ij}^{\mathrm{MAX}}/Pr_{ij}^{\mathrm{MIN}})}{\ln(1+\varepsilon)}\rceil. \tag{22}$$

For example, we set $n_i = 3$ in Fig. 5. Thus, the discretization interval is partitioned into 3 subintervals, and any angles in the same subinterval have the same approximate charging power.

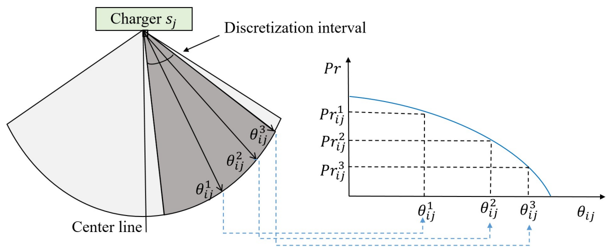

Fig. 5. Angle discretization with $n_i = 3$. The black dotted curves stand for the approximated value of charging power.

Let $Pr(\theta_{ij})$ denote the charging power that MRSD o_i receives from s_j with charging angle θ_{ij}. Given the discretization error $\varepsilon > 0$, we have the following lemma.

Lemma 6. *Let $Pr_{ij}^0 = Pr_{ij}^{\mathrm{MAX}}$, $Pr_{ij}^{n_i} = Pr_{ij}^{\mathrm{MIN}}$, and $Pr_{ij}^v = Pr_{ij}^{v-1}(1+\varepsilon)^{-1}$ for $v = 1, 2, ..., n_i - 1$ (therefore, $\theta_{ij}^v = cos^{-1}(\frac{Pr_{ij}^v(\beta+d_j)^2}{\mu} - \alpha)$ and $n_i = \lceil \frac{\ln(Pr_{ij}^{\mathrm{MAX}}/Pr_{ij}^{\mathrm{MIN}})}{\ln(1+\varepsilon)} \rceil$). For any $|\theta_{ij}| \leq \delta$, let $\widetilde{\theta}_{ij}$ be the corresponding discrete angle of θ_{ij}, we have*

$$\frac{Pr(\theta_{ij})}{Pr(\widetilde{\theta}_{ij})} \leq 1 + \varepsilon. \tag{23}$$

Proof: If $\theta_{ij} = cos^{-1}(\frac{Pr_{ij}^{\mathrm{MAX}}(\beta+d_j)^2}{\mu} - \alpha)$, we have $\frac{Pr(\theta_{ij})}{Pr(\widetilde{\theta}_{ij})} = \frac{Pr_{ij}^{\mathrm{MAX}}}{Pr_{ij}^1} = 1 + \varepsilon$.

If $\theta_{ij}^{v-1} < \theta_{ij} \leq \theta_{ij}^v$, we have $\frac{Pr(\theta_{ij})}{Pr(\widetilde{\theta}_{ij})} = \frac{Pr(\theta_{ij})}{Pr(\theta_{ij}^v)} \leq \frac{Pr(\theta_{ij}^{v-1})}{Pr(\theta_{ij}^v)} = \frac{Pr_{ij}^{v-1}}{Pr_{ij}^v} = 1 + \varepsilon$. ■

In order to obtain the finite candidate angles of MRSD with the largest actual replenished energy, in theory, we should conduct angle discretization for all MRSDs in the charging group. This is because we need to guarantee that every MRSD in the charging group can obtain the approximate charging power. To further improve the search efficiency, we show that it is sufficient to bound the error by conducting angle discretization for only two MRSDs in the charging group.

Lemma 7. *Given the charging group $G_j = \{o_1, o_2, ..., o_k\}$, $E_1(s_j) \geq E_2(s_j) \geq ... \geq E_k(s_j)$, $k > 1$, conducting angle discretization for o_{k-1} and o_k, we have*

$$\frac{Pr(\theta_{ij})}{Pr(\widetilde{\theta}_{ij})} \leq 1 + \varepsilon, \forall o_i \in G_j. \tag{24}$$

Proof: We first consider the anticlockwise side of charger's center line. Let $\vec{G}_j$ be the set of MRSDs on the anticlockwise side of charger's center line. Given $Pr(\theta_{ij}) = \frac{\mu(cos\theta_{ij}+\alpha)}{(\beta+d_j)^2}$, $\theta_{ij} \in (0, \delta]$, $\delta \in (0, \frac{\pi}{2}]$. The first order derivative of $Pr(\theta_{ij})$ is: $Pr'(\theta_{ij}) = \frac{-\mu sin\theta_{ij}}{(\beta+d_j)^2} < 0$. The second derivative of $Pr(\theta_{ij})$ is: $Pr''(\theta_{ij}) = \frac{-\mu cos\theta_{ij}}{(\beta+d_j)^2} < 0$. Therefore, $Pr(\theta_{ij})$ is a strictly decreasing concave function of θ_{ij} for $\theta_{ij} \in (0, \delta]$, $\delta \in (0, \frac{\pi}{2}]$. For any $\delta \geq \theta_{i'j} + \triangle\theta = \widetilde{\theta}_{i'j} > \theta_{i'j} \geq \theta_{ij} > 0$, we have

$$Pr(\theta_{ij}) - Pr(\theta_{ij} + \triangle\theta) < Pr(\theta_{i'j}) - Pr(\theta_{i'j} + \triangle\theta). \tag{25}$$

When $\frac{Pr(\theta_{i'j})}{Pr(\widetilde{\theta}_{i'j})} = \frac{Pr(\theta_{i'j})}{Pr(\theta_{i'j} + \triangle\theta)} \leq 1 + \varepsilon$, we have

$$Pr(\theta_{i'j}) - Pr(\theta_{i'j} + \triangle\theta) \leq \varepsilon Pr(\theta_{i'j} + \triangle\theta). \tag{26}$$

Since $Pr(\theta_{ij})$ is a strictly decreasing function of θ_{ij} for $\theta_{ij} \in (0, \delta]$, $\delta \in (0, \frac{\pi}{2}]$, we have

$$\varepsilon Pr(\theta_{i'j} + \triangle\theta) < \varepsilon Pr(\theta_{ij} + \triangle\theta). \tag{27}$$

Integrating (25), (26) and (27), we have

$$\frac{Pr(\theta_{ij})}{Pr(\theta_{ij} + \triangle\theta)} \leq 1 + \varepsilon, \forall o_i \in \vec{G}_j. \tag{28}$$

This indicates that conducting angle discretization for o_i' can guarantee the discretization error of o_i.

For the situation of clockwise side of charger's center line, it is not difficult to get the similar conclusion using the same deduction. ■

Lemma 7 indicates that we only need to conduct angle discretization for the farthest MRSDs on both sides (i.e., o_{k-1} and o_k, $k > 1$). The discretization intervals of o_{k-1} and o_k can be calculated using Lemma 3 and Lemma 5 through mapping the discretization interval of o_1 to o_{k-1} and o_k. Specifically, we consider the following two cases:

Case 1: k is odd.

The discretization intervals of o_{k-1} and o_k are

$$\theta_{k-1,j} \in [\frac{1-k}{2}\theta_j^{\triangle}, \min\{(1-\frac{k}{2})\theta_j^{\triangle}, \delta-(k-1)\theta_j^{\triangle}\}]. \tag{29}$$

$$\theta_{k,j} \in [\frac{k-1}{2}\theta_j^{\triangle}, \min\{\frac{k}{2}\theta_j^{\triangle}, \delta\}]. \tag{30}$$

Case 2: k is even.

The discretization intervals of o_{k-1} and o_k are

$$\theta_{k-1,j} \in [\max\{(\frac{k}{2}-1)\theta_j^{\triangle}, (k-1)\theta_j^{\triangle}-\delta\}, \min\{\frac{k-1}{2}\theta_j^{\triangle}, \delta\}]. \tag{31}$$

$$\theta_{k,j} \in [\max\{-\frac{k}{2}\theta_j^{\triangle}, -\delta\}, \min\{\frac{1-k}{2}\theta_j^{\triangle}, \delta-(k-1)\theta_j^{\triangle}\}]. \tag{32}$$

Let $\widetilde{\phi}_j$ be the approximate arrangement, we have the following lemma:

Lemma 8. *Given the charging group $G_j = \{o_1, o_2, ..., o_k\}$, $E_1(s_j) \geq E_2(s_j) \geq ... \geq E_k(s_j)$, $k > 1$, conducting angle discretization for o_{k-1} and o_k, we have*

$$\frac{T(G_j, \widetilde{\phi}_j)}{T(G_j, \phi_j)} \leq 1 + \varepsilon. \tag{33}$$

Algorithm 1 : Charging Scheduling Algorithm (CSA)

Input: M, N, α, β, μ, δ, E_i, b_i, $\forall o_i \in N$, $\theta_j^{\triangle}$, a_j, A_j, T_j, d_j, $\forall s_j \in M$
Output: $(G_j, \phi_j), \forall s_j \in M$
1: **foreach** $s_j \in M$ **do**
2: $G_j \leftarrow \emptyset$; $\phi_j \leftarrow \emptyset$;
3: **end**
4: $N' \leftarrow N$;
5: **while** $N' \neq \emptyset$ **do**
6: **foreach** $s_j \in M$ **do**
7: $(G'_j, \phi'_j) \leftarrow \text{CEM}(s_j, G_j, N')$;
8: **end**
9: $s_j \leftarrow \arg \min_{s_{j'} \in M, G_{j'} \subset G'_{j'}} \dfrac{c(G'_{j'}, \phi'_{j'}) - c(G_{j'}, \phi_{j'})}{|G'_{j'} \backslash G_{j'}|}$;
10: $G_j \leftarrow G'_j$; $\phi_j \leftarrow \phi'_j$; $N' \leftarrow N' \backslash G'_j$;
11: **end**

Algorithm 2 : Cost Effectiveness Minimization (CEM)

Input: s_j, G_j, N'
Output: (G'_j, ϕ'_j)
1: $l \leftarrow 0$; $G_j(l) \leftarrow G_j$; $N^* \leftarrow N'$;
2: sort o_i based on $E_i(s_j)$ for $\forall o_i \in G_j$ in the nonincreasing order and the sequence is denoted by $Q_j \leftarrow \{o_1, o_2, ..., o_{|G_j|}\}$;
3: **while** $l + |G_j| < \lfloor \dfrac{2\delta}{\theta_j^{\triangle}} \rfloor + 1$ **and** $l < |N'|$ **do**
4: $o_i \leftarrow \arg \min_{o_{i'} \in N^*} E_{i'}(s_j)$;
5: $N^* \leftarrow N^* \backslash \{o_i\}$;
6: $l \leftarrow l + 1$; $G_j(l) \leftarrow G_j(l-1) \cup \{o_i\}$; $Q_j \leftarrow Q_j \overset{+}{\cup} \{o_i\}$;
7: $\widetilde{\phi}_j(l) \leftarrow \text{AD}(Q_j, G_j(l), l)$;
8: **end**
9: $(G'_j, \phi'_j) \leftarrow \arg \min_{(G_j(l), \widetilde{\phi}_j(l)):l>0} \dfrac{c(G_j(l), \widetilde{\phi}_j(l)) - c(G_j, \phi_j)}{l}$;

Algorithm 3 : Angle Discretization (AD)

Input: Q_j, $G_j(l)$, l
Output: $\widetilde{\phi}_j(l)$
1: $k \leftarrow |G_j(l)|$;
2: **if** $k == 1$ **then**
3: $\widetilde{\phi}_j(l) \leftarrow \{0°\}$;
4: **return** $\widetilde{\phi}_j(l)$;
5: **end**
6: **if** k is odd **then**
7: calculate the discretization interval of o_{k-1} in Q_j based on (29);
8: calculate the discretization interval of o_k in Q_j based on (30);
9: **else**
10: calculate the discretization interval of o_{k-1} in Q_j based on (31);
11: calculate the discretization interval of o_k in Q_j based on (32);
12: **end**
13: conduct angle discretization for o_{k-1} and o_k;
14: let $\Psi_j(l)$ be the set of all candidate arrangements after angle discretization;
15: $\widetilde{\phi}_j(l) \leftarrow \arg \min_{\widetilde{\phi'}_j(l) \in \Psi_j(l)} T(G_j(l), \widetilde{\phi'}_j(l))$;

Proof: We first consider that the MRSD with the maximum charging time is on the anticlockwise side of charger's center line. We have

$$\frac{T(G_j, \widetilde{\phi}_j)}{T(G_j, \phi_j)} = \frac{\max_{o_i \in G_j} T_i(s_j, \widetilde{\theta}_{ij})}{\max_{o_i \in G_j} T_i(s_j, \theta_{ij})} = \frac{\max_{o_i \in \vec{G}_j} \dfrac{E_i(s_j)}{Pr(\widetilde{\theta}_{ij})}}{\max_{o_i \in \vec{G}_j} \dfrac{E_i(s_j)}{Pr(\theta_{ij})}}$$

$$\leq \frac{\max_{o_i \in \vec{G}_j} \dfrac{E_i(s_j)}{Pr(\widetilde{\theta}_{ij})}}{\max_{o_i \in \vec{G}_j} \dfrac{E_i(s_j)}{(1+\varepsilon)Pr(\widetilde{\theta}_{ij})}} = 1 + \varepsilon$$

where the inequality relies on (24).

For the situation that the MRSD with maximum charging time is on the clockwise side of charger's center line, it is not difficult to get the similar conclusion using the same deduction. ■

After angle discretization, the CCSS problem can be reformulated as

$$(\widetilde{CCSS}): \quad \min \sum_{s_j \in M} c(G_j, \widetilde{\phi}_j), \tag{34}$$

$$s.t. \quad \bigcup_{s_j \in M} G_j = N, \tag{34-1}$$

$$G_j \cap G_{j'} = \emptyset, \ \forall s_j \neq s_{j'}, \ s_j \in M, \ s_{j'} \in M, \tag{34-2}$$

$$|\widetilde{\theta}_{ij} - \widetilde{\theta}_{i'j}| \geq \theta_j^{\triangle}, \ \forall o_i \neq o_{i'}, \ o_i, o_{i'} \in G_j, s_j \in M, \tag{34-3}$$

$$|\widetilde{\theta}_{ij}| \leq \delta, \ \forall o_i \in G_j, \ s_j \in M. \tag{34-4}$$

4.5 Algorithm Design

CSA outputs the charging group G_j and the arrangement of MRSDs ϕ_j for each charger $s_j \in M$. As illustrated in Algorithm 1, we call the function CEM($\cdot$) (Line 7) to find the MRSD set from the unassigned MRSD set N' and the corresponding arrangement ϕ'_j for each s_j that can minimize the cost effectiveness over s_j's current charging group G_j after discretization. Then we find the extended charging group G'_j with minimum cost effectiveness among all chargers (Line 9), and update the charging group, arrangement of MRSDs of s_j and the unassigned MRSD set N'(Lines 10). The iteration terminates when all MRSDs are assigned.

As illustrated in Algorithm 2, the function CEM($\cdot$) returns the charging group and the corresponding arrangement with the minimum cost effectiveness. Let l be the number of MRSDs added into the charging group. Let $G_j(l)$ be the charging group of s_j after l MRSDs are added. We first sort all MRSDs in the current charging group based on the actual replenished energy in the nonincreasing order, and the sequence is denoted by Q_j (Line 2). Then, we traverse all possible values of l. If the charger still has space for placing MRSDs and there are unassigned MRSDs (Line 3), we find the MRSD with lowest actual replenished energy from the unassigned MRSD set N^* (Line 4), and assign it to the charging group and MRSD queue (Line 6), where operation $\overset{+}{\cup}$ represents inserting MRSD in front of the queue. Note that we always select the MRSD with lowest actual replenished energy from the unassigned MRSD set at each iteration, therefore, the selected MRSD must be in the front of the queue. Then, we call function AD $(\cdot)$ to find the near optimal arrangement $\widetilde{\phi}_j(l)$ through angle discretization (Line 7). Finally, we return the extended charging group and the corresponding arrangement with the minimum cost effectiveness from all $G_j(l)$ and $\widetilde{\phi}_j(l)$ for possible values of l, $l=1,2,...,\min\{\lfloor\frac{2\delta}{\theta_j^{\triangle}}\rfloor+1-|G_j|,|N'|\}$ (Line 9).

The function AD $(\cdot)$ illustrated in Algorithm 3 returns the near optimal arrangement $\widetilde{\phi}_j(l)$ of charger s_j through angle discretization when l MRSDs have added in the charging group. Let k be the number of MRSDs in $G_j(l)$ (Line 1). If k is odd, we calculate the discretization intervals of the $(k-1)$-th MRSD and the k-th MRSD in the queue Q_j based on (29) and (30), respectively (Lines 7-8). If k is even, we calculate the discretization intervals of the $(k-1)$-th MRSD and the k-th MRSD in the queue Q_j based on (31) and (32), respectively (Lines 10-11). For each discretization interval, we conduct angle discretization (Line 13), which has been given in Lemma 6. Each discrete angle of o_{k-1} and o_k corresponds to a candidate arrangement of $G_j(l)$. We denote the set of all candidate arrangements after angle discretization by $\Psi_j(l)$ (Line 14). Finally, we find the arrangement with minimum charging time of charging group $G_j(l)$ from $\Psi_j(l)$ (Line 15). Note that, given the value of l, minimizing the cost effectiveness is equivalent to minimizing the charging time of extended charging group.

Next, we give the time complexity analysis and approximation analysis of CSA.

Lemma 9. *The time complexity of CSA is* $O\left(n^3m\left(\left\lceil\frac{\ln\frac{1+\alpha}{cos\delta+\alpha}}{\ln(1+\varepsilon)}\right\rceil+1\right)\right)$.

Proof: We first analyze the time complexity of AD$(\cdot)$. The running time of AD$(\cdot)$ is dominated by finding the arrangement with minimum charging time of charging group (Line 15 of Algorithm 3). According to Lemma 6, the number of discrete angles of any $o_i\in G_j$ is $n_i+1=\left\lceil\frac{\ln\left(Pr_{ij}^{MAX}/Pr_{ij}^{MIN}\right)}{\ln(1+\varepsilon)}\right\rceil+1$. Considering the largest discretization interval $[0,\delta]$, the total number of discrete angles of o_{k-1} and o_k is $2\left(\left\lceil\frac{\ln\frac{1+\alpha}{cos\delta+\alpha}}{\ln(1+\varepsilon)}\right\rceil+1\right)$. For each $\widetilde{\phi}'_j(l)$, calculating $T(G_j(l),\widetilde{\phi}'_j(l))$ tasks at most $O(n)$ time. Therefore, AD$(\cdot)$ takes $O\left(n\left(\left\lceil\frac{\ln\frac{1+\alpha}{cos\delta+\alpha}}{\ln(1+\varepsilon)}\right\rceil+1\right)\right)$ time. The time complexity of CEM$(\cdot)$ is dominated by executing AD$(\cdot)$ (Line 7 of Algorithm 2), which takes $O\left(n^2\left(\left\lceil\frac{\ln\frac{1+\alpha}{cos\delta+\alpha}}{\ln(1+\varepsilon)}\right\rceil+1\right)\right)$ time. In CSA, CEM$(\cdot)$ is executed at most nm times. Therefore, the time complexity of CSA is $O\left(n^3m\left(\left\lceil\frac{\ln\frac{1+\alpha}{cos\delta+\alpha}}{\ln(1+\varepsilon)}\right\rceil+1\right)\right)$. ■

<u>*Remark*</u>: Note that the running time of CSA, $O\left(n^3m\left(\left\lceil\frac{\ln\frac{1+\alpha}{cos\delta+\alpha}}{\ln(1+\varepsilon)}\right\rceil+1\right)\right)$, is very conservative since the discretization interval is much small than $[0,\delta]$ in practice.

Theorem 2. *CSA is a* $(\ln n+1)(1+\varepsilon)$*-approximation algorithm of the CCSS problem.*

Proof: We first prove CSA is a $(\ln n+1)$-approximation algorithm of the $\widetilde{CCSS}$ problem. Number the MRSDs of N in the order in which they were covered by CSA resolving ties arbitrarily. Let o_1, o_2, o_3, ..., o_n be the numbering. Assume o_k, $k=1,2,...,n$ is covered by the extended charging group G'_j of charger s_j over G_j. Then the cost effectiveness of o_k is defined as

$$cost(o_k)=\frac{c(G'_j,\phi'_j)-c(G_j,\phi_j)}{|G'_j\backslash G_j|}. \tag{35}$$

Let $\widetilde{OPT}$ be the optimal total cost of the $\widetilde{CCSS}$ problem. Consider the iteration in which o_k was covered, the charging groups of optimal solution can cover the remaining MRSDs in N' with cost at most $\widetilde{OPT}$. Therefore, among all charging groups in the optimal solution, there must be one having cost effectiveness at most $\widetilde{OPT}/|N'|$, where $|N'|\geq n-k+1$. Since o_k was covered by set G'_j of charger s_j with minimum cost effectiveness in this iteration, it follows

$$cost(o_k)\leq\frac{\widetilde{OPT}}{|N'|}\leq\frac{\widetilde{OPT}}{n-k+1}. \tag{36}$$

Since the cost of each charging group is distributed among the new MRSDs covered, the total cost of the charging groups obtained by CSA is equal to

$$\begin{aligned}&\textstyle\sum_{k=1}^{n}cost(o_k)\\&\leq\textstyle\sum_{k=1}^{n}\frac{\widetilde{OPT}}{n-k+1}\\&=(1+\frac{1}{2}+...+\frac{1}{n})\widetilde{OPT}\\&\leq(\ln n+1)\widetilde{OPT}.\end{aligned} \tag{37}$$

Next, we denote the optimal total cost of the CCSS problem by OPT. We show that $\widetilde{OPT}\leq(1+\varepsilon)OPT$. We consider the following three cases:

Case 1: $T(G_j,\phi_j)\leq T(G_j,\widetilde{\phi}_j)\leq T_j$.
Based on (5), we have

$$c(G_j,\widetilde{\phi}_j)=c(G_j,\phi_j)=A_j. \tag{38}$$

Case 2: $T(G_j,\phi_j)\leq T_j<T(G_j,\widetilde{\phi}_j)$.
We have

$$\begin{aligned}&\frac{c(G_j,\widetilde{\phi}_j)}{c(G_j,\phi_j)}\\&=\frac{A_j+a_j(T(G_j,\widetilde{\phi}_j)-T_j)}{A_j}\\&\leq1+\frac{a_j(T(G_j,\widetilde{\phi}_j)-T_j)}{a_jT_j}\\&=\frac{T(G_j,\widetilde{\phi}_j)}{T_j}\\&\leq\frac{T(G_j,\widetilde{\phi}_j)}{T(G_j,\phi_j)}\\&\leq1+\varepsilon,\end{aligned} \tag{39}$$

where the first inequality relies on (6), the second inequality relies on $T(G_j,\phi_j)\leq T_j$, and the last inequality relies on (33).

Case 3: $T_j<T(G_j,\phi_j)\leq T(G_j,\widetilde{\phi}_j)$.

We have

$$\begin{aligned}&\frac{c(G_j,\widetilde{\phi}_j)}{c(G_j,\phi_j)}\\&=\frac{A_j+a_j(T(G_j,\widetilde{\phi}_j)-T_j)}{A_j+a_j(T(G_j,\phi_j)-T_j)}\\&\le\frac{A_j+a_j((1+\varepsilon)T(G_j,\phi_j)-T_j)}{A_j+a_j(T(G_j,\phi_j)-T_j)}\\&=1+\frac{a_j\varepsilon T(G_j,\phi_j)}{A_j+a_j(T(G_j,\phi_j)-T_j)}\\&\le 1+\varepsilon,\end{aligned} \tag{40}$$

where the first inequality relies on (33), and the last inequality relies on (6).

Based on (38), (39) and (40), we have:

$$\frac{\widetilde{OPT}}{OPT}=\frac{\sum_{s_j\in M}c(G_j,\widetilde{\phi}_j)}{\sum_{s_j\in M}c(G_j,\phi_j)}\le 1+\varepsilon. \tag{41}$$

Combining (37) and (41), we obtain the theorem. ■

5 Simulation Results

In this section, we perform simulations to verify the performance of our algorithm.

5.1 Simulation Setup

For the simulations, we randomly distribute the chargers and MRSDs in a 2D plane. The default values of parameters are given in Table 2. The unit of power is watt. We will vary the value of the key parameters to explore the impacts on the algorithms. All the simulations are run on a Windows machine with Intel(R) Xeon(R) CPU i5-8250U and 8 GB memory. Each measurement is averaged over 100 instances.

TABLE 2
Default Settings of Parameters

Parameter	Default value
Ω	$250m * 250m$
m	30
n	100
δ	$\pi/2$
x	$0.2m$
E_i	$[1500, 2000]J$
b_i	$[5, 10]J/m$
a_j	$[0.001, 0.0015]$
A_j	$[200, 220]$
T_j	$[60000, 80000]s$
d_j	$[0.4, 0.6]m$
α, β, μ	0.11, 0.01, 0.0108
ε	0.1

We compare our algorithm with following three algorithms:

- **IBC (Improved BC)**: We modify the BC in [10] to fit the scenario of this paper. In each iteration, IBC traverses all MRSDs, and each MRSD chooses the best available charger (space feasible) and charging angle such that the increased cost is minimized. Then the MRSD with the lowest marginal cost is assigned. The charging angles of assigned MRSDs no longer change in the subsequent iterations. The iterations terminate when all MRSDs are assigned.
- **GBC (Group BC)**: GBC is also modified from the BC in [10]. In each iteration, GBC traverses all chargers. For each charger, GBC finds a set of unsigned MRSDs to form a charging group such than the cost of charging group is minimized. The MRSD with maximum actual replenished energy is placed at the center line. The arrangement of other MRSDs follows the properties given in section 4.3. Then the charger with the lowest cost of charging group is selected. The iterations terminate when all MRSDs are assigned.
- **IAASA (Improved AASA)**: We modify the AASA in [37] to fit the scenario in this paper. The only difference with GBC is that the charger with the lowest average cost (the ratio of charging cost to the size of group) is selected at the end of each iteration.

5.2 Total Cost

To test the scalability of our algorithm, we increase the number of MRSDs from 70 to 120. As shown in Fig. 6, the total cost of all algorithms increases with the increasing number of MRSDs. CSA always outputs the lowest total cost. Specifically, CSA reduces 37.7%, 10.9% and 10.0% of total cost on average comparing with IBC, GBC and IAASA, respectively. IBC selects the MRSD with minimum marginal cost at each time, and allocates the best charging angle to the currently selected MRSD. Therefore, the MRSD with low actual replenished energy will obtain the high power, increasing the total cost. On the other hand, both GBC and IAASA determine a set of MRSDs to form the charging group at a time rather than assign a single MRSD to the charger, thus, some of MRSDs in the charging group probably do not suit for the current charger. Moreover, GBC and IAASA do not conduct the angle discretization, therefore, cannot bound the performance.

Then, we increase the number of chargers from 20 to 45. As shown in Fig. 7, the total cost of all algorithms decreases with the increasing number of chargers. This is because with more chargers, the MRSDs can move to closer or cheaper chargers. Averagely, CSA reduces the total cost by 37.7%, 12.3% and 10.2% comparing with IBC, GBC and IAASA, respectively.

We change the length of MRSDs. With the increasing spatial occupation of MRSD, less MRSDs can be placed in the charging groups. This indicates that the cooperation opportunities reduce and more chargers are needed. Therefore, the total cost of all algorithms increases. As shown in Fig. 8, CSA reduces 42.5%, 15.6% and 15.0% of total cost on average comparing with IBC, GBC and IAASA, respectively.

Fig. 9 shows the impact of base fare on the total cost. With the increase in base fare, the cost of opening new charging group increases, therefore, the total cost of all algorithms increase accordingly. Moreover, we can see that the total cost of IBC increases rapidly than other three algorithms. This is because IBC chooses the best available charging angle for the current MRSD such that the increased cost is minimized. Once the charging angle is determined, IBC does not change the charging angle in the subsequent iterations. Thus, the average spatial occupation of MRSDs is

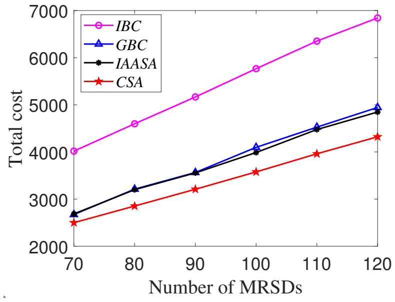

Fig. 6. Total cost vs. n

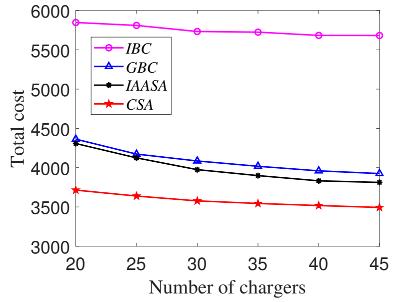

Fig. 7. Total cost vs. m

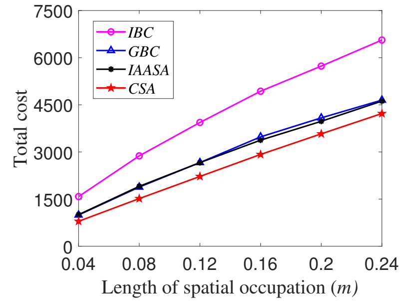

Fig. 8. Total cost vs. x

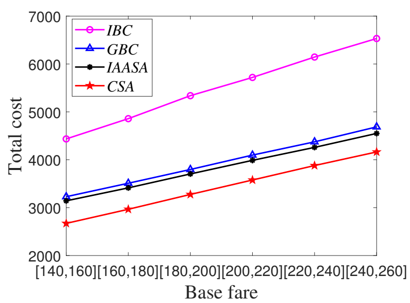

Fig. 9. Total cost vs. base fare

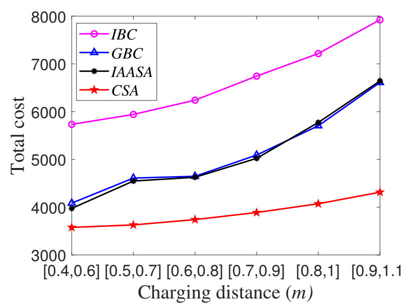

Fig. 10. Total cost vs. charging distance

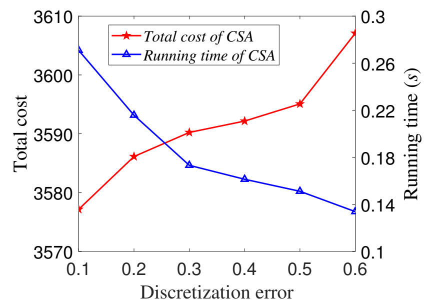

Fig. 11. Total cost and running time vs. ε

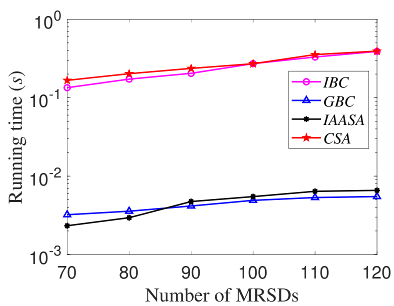

Fig. 12. Running time vs. n

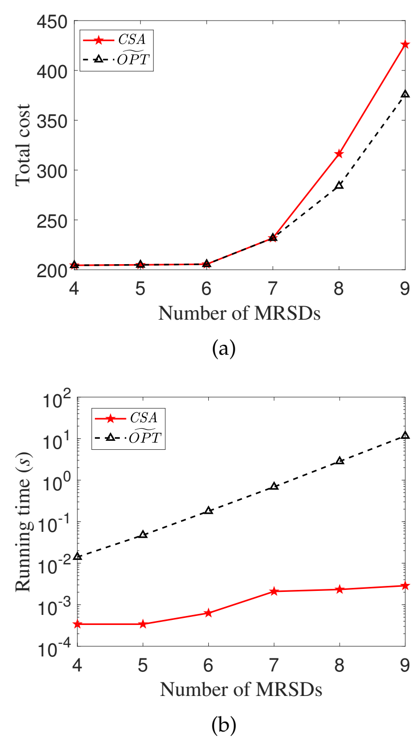

Fig. 13. Comparison with the optimal solution of $\widetilde{CCSS}$ problem. (a) total cost (b) running time.

larger than other algorothms, and more chargers are needed in IBC. Averagely, CSA reduces the total cost by 37.9%, 13.6% and 11.2% comparing with IBC, GBC and IAASA, respectively.

Fig. 10 shows the impact of charging distance on the total cost. With the increase in charging distance, the total cost of all algorithms increases. This is because the charging power decreases with increasing charging distance, thus, more charging time is needed to fulfill the energy demands of MRSDs. Averagely, CSA reduces the total cost by 41.3%, 23.4% and 22.8% comparing with IBC, GBC and IAASA, respectively.

Fig. 11 shows the impact of angle discretization error on the cost of CSA. With the increase in discretization error, the total cost of CSA increases accordingly. When $\varepsilon = 0.6$, the total cost increases by 29.93 comparing with the total cost when $\varepsilon = 0.1$. This consistent with our approximation analysis given in Theorem 2.

5.3 Running Time

We measure the running time of IBC, GBC, IAASA and CSA. We can see from Fig. 12 that the running time of all algorithms grows with increasing number of MRSDs. The running time of GBC and IAASA is much lower than IBC and CSA. This is because GBC and IAASA assign MRSDs group by group, while IBC and CSA assign MRSDs one by one. The running time of CSA is higher than IBC since CSA conduct angle discretization. However, CSA can output the solution in 0.4 seconds when there are 120 MRSDs. As shown in Fig. 11, the running time of CSA decreases sharply with the increase in discretization error. This is because less discrete charging angles need to be traversed in Algorithm 3.

5.4 Comparison with $\widetilde{OPT}$

Since the possible charging angles are infinite, we compare the performance of our algorithms with the optimal solution of $\widetilde{CCSS}$ problem in a small charging network ($40m*40m$). To realize the $\widetilde{OPT}$, we traverse all possible assigments between MRSDs and chargers. Given the MRSDs of each charging group, we adopt the optimal arrangement and angle discretization presented in section 4.3 and section 4.4, respectively. As shown in Fig. 13, the total cost of CSA is only 4.12% higher than that of $\widetilde{OPT}$ on average. At the beginning, the results of the two algorithms are exactly same because when the number of MRSDs is small, one charger can meet all charging demands. For the same charging group, both algorithms determine the charging arrangement in the same way. However, when the number of MRSDs increases, more chargers are needed, and CSA can only obtain approximate solutions. However, $\widetilde{OPT}$ takes 11.6 seconds even for 9 MRSDs, and is much slower than CSA.

6 FIELD EXPERIMENTS

We have conducted the field experiments to evaluate all five algorithms. We implemented the algorithms on a testbed consisting of 8 MRSDs, 5 directional chargers (TX91501 power transmitters produced by Powercast [38]), and an AP that connects to a laptop for reporting energy data collected from the MRSDs as shown in Fig. 14. We carried out the experiment in $40m * 40m$ square area. The coordinates of the chargers are (10,10), (10, 30), (20, 20), (30, 10), and (30,30). The 8 MRSDs are randomly placed in the area. According to our tests, we have $\alpha = 0.11$, $\beta = 0.01$, and $\mu = 0.00738$. Since the charging time will reach tens of hours in previous simulations, we change the parameter settings in order to accelerate the field experiments. In our field experiments, the energy demands of MRSDs, base fare and charging time threshold of chargers are in $[150, 200]$, $[20, 22]$ and $[6000, 8000]$, respectively. Moreover, such parameter settings make the charging time of some MRSDs lower than the time threshold and the charging time of others higher than the time threshold, which helps to simulate a real world situation with various charging demands. The specific parameter settings of chargers and MRSDs are summarized in Table 3 and Table 4, respectively. The settings of other parameters are same with those in Table 2.

The scheduling results of CSA are shown in Fig. 15, where the triangles represent the chargers and the dots represent the MRSDs. The arrangment of MRSDs of charging group G_5 of CSA is illustrated in Fig. 16. The quantized scheduling results and total cost of all algorithms are summarized in Table 4. The scheduling result of each MRSD is represented as a two-tuple, where the first item represents the charger assigned and the second item represents the charging angle. We can see that CSA outputs the same scheduling result with $\widetilde{OPT}$, and only need one charger due to the fine-grained arrangement of MRSDs through the angle discretization. CSA reduces 36.3% and 22.0% total cost comparing with IBC and GBC (IAASA) in our field experiments, respectively.

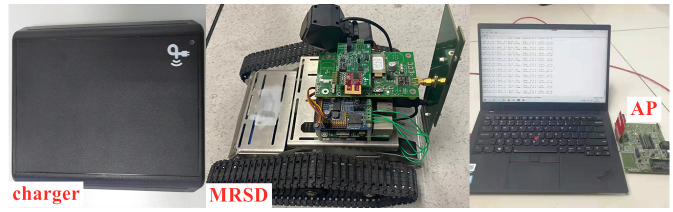

Fig. 14. Testbed

TABLE 3
Parameters of All Chargers

Charger	A_j	a_j	$T_j(s)$	$d_j(m)$
1	21.7	0.0011	6240	0.45
2	20.4	0.0012	7250	0.55
3	21.3	0.0014	6690	0.5
4	21.8	0.0013	6670	0.5
5	20.4	0.0011	7150	0.5

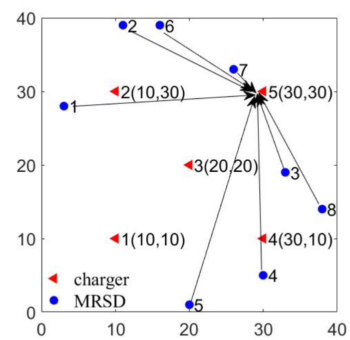

Fig. 15. Scheduling results of CSA

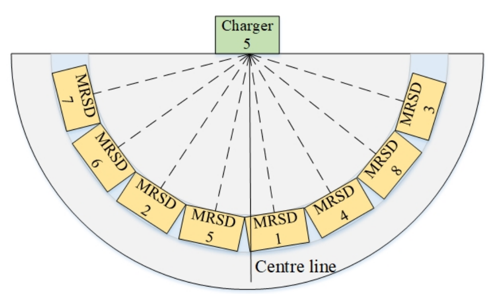

Fig. 16. Charging group G_5 of CSA

TABLE 4
Scheduling Results of All Algorithms

MRSD	$E_i(J)$	$b_i(m/J)$	IBC	GBC, IAASA	CSA, $\widetilde{OPT}$
1	199	8.7	$(2, 0°)$	$(2, 0°)$	$(5, 7.7°)$
2	176	9.7	$(2, -20.6°)$	$(5, 22.6°)$	$(5, -37.5°)$
3	171	7.5	$(2, 20.6°)$	$(5, -67.8°)$	$(5, 75.5°)$
4	160	8.9	$(2, -41.2°)$	$(5, -22.6°)$	$(5, 30.3°)$
5	166	7.3	$(2, 41.2°)$	$(5, 0°)$	$(5, -14.9°)$
6	155	9.2	$(2, -61.8°)$	$(5, 45.2°)$	$(5, -60.1°)$
7	169	6.9	$(2, 61.8°)$	$(5, 67.8°)$	$(5, -82.7°)$
8	166	8.7	$(4, 0°)$	$(5, -45.2°)$	$(5, 52.9°)$
Total cost			81.3	66.4	51.8

7 Discussion

Our CSA can also be used to solve the CCSS problem in omnidirectional wireless charging case after a minor modification. In omnidirectional wireless charging, the charging power from any charger s_j to any MSRD o_i is

$$Pr(s_j, o_i) = \begin{cases} \dfrac{\alpha}{(\beta + d_j)^2}, & d_j \le D \\ 0, & otherwise. \end{cases} \tag{42}$$

For any charger $s_j \in M$, at most $\lfloor \frac{2\pi}{\theta_j^{\triangle}} \rfloor$ MRSDs can be placed in the charging group G_j. Thus, the following inequation should be satisfied to guarantee that there is enough space to place all MRSDs:

$$\sum_{s_j \in M} \lfloor \frac{2\pi}{\theta_j^{\triangle}} \rfloor \ge n. \tag{43}$$

The CCSS problem for omnidirectional wireless charging can be formulated as

$$\min \sum_{s_j \in M} c(G_j), \tag{44}$$

$$s.t. \bigcup_{s_j \in M} G_j = N, \tag{44-1}$$

$$G_j \cap G_{j'} = \emptyset, \ \forall s_j \ne s_{j'}, \ s_j \in M, \ s_{j'} \in M, \tag{44-2}$$

$$|G_j| \le \lfloor \frac{2\pi}{\theta_j^{\triangle}} \rfloor, \forall s_j \in M. \tag{44-3}$$

This problem is still NP-hard by reduction from the CGFLP problem, where the capacity of facility s_j is $\lfloor \frac{2\pi}{\theta_j^{\triangle}} \rfloor$.

The work flow of CSA for omnidirectional wireless charging is similar with that for directional wireless charging. The difference is that the charging angle calculation and angle discretization (Algorithm 3) are not needed. We can determine the charging group with the minimum cost effectiveness for any charger s_j directly. Since Algorithm 3 is not needed, Algorithm 2 takes $O(n^2)$ time, and the time complexity of CSA for omnidirectional wireless charging is $O(n^3 m)$. Moreover, since it is not necessary to conduct angle discretization, CSA for omnidirectional wireless charging can achieve $(\ln n + 1)$-approximation for the CCSS problem in omnidirectional wireless charging case.

8 Conclusion

In this paper, we have presented a cooperative charging system model for directional wireless charging with spatial occupation and have formulated the CCSS problem for optimizing the total charging cost. The properties of optimal arrangement of MRSDs in the charging group has been revealed. We have employed the angle discretization technique to get the finite candidate charging angles and the corresponding approximate charging power of MRSDs. We also have shown that it is sufficient to bound the error by conducting angle discretization for only two MRSDs in the charging group. We have proposed a $(\ln n + 1)(1 + \varepsilon)$-approximation algorithm based on the greedy approach. The results demonstrate that our algorithm can reduce at most 42.5% and 36.3% total cost comparing with the benchmark algorithms in extensive simulations and field experiments, respectively.

Acknowledgments

This work has been supported in part by the National Natural Science Foundation of China (No. 61872193, 61872191, 62072254) and the Graduate Student Scientific Research Innovation Projects in Jiangsu Province (KYCX22-1025) .

References

[1] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han, "Wireless Charging Technologies: Fundamentals, Standards, and Network Applications," *IEEE Communications Surveys & Tutorials*, vol. 18, no. 2, pp. 1413-1452, 2016.

[2] P. Zhou, C. Wang and Y. Yang, "Self-sustainable Sensor Networks with Multi-source Energy Harvesting and Wireless Charging," in *IEEE INFOCOM*. IEEE, 2019, pp. 1828-1836.

[3] Y. Jin, J. Xu, S. Wu, L. Xu, D. Yang, K. Xia. "Bus Network Assisted Drone Scheduling for Sustainable Charging of Wireless Rechargeable Sensor Network," *Journal of Systems Architecture*, vol. 116, 2021, doi:10.1016/j.sysarc.2021.102059.

[4] Y. Jin, J. Xu, S. Wu, L. Xu, D. Yang, "Enabling the Wireless Charging via Bus Network: Route Scheduling for Electric Vehicles," *IEEE Transactions on Intelligent Transportation Systems*, vol. 22, no. 3, pp. 1827-1839, 2021.

[5] H. Liu, X. Huang, L. Tan, J. Guo, W. Wang, C. Yan, and C. Xu, "Dynamic Wireless Charging for Inspection Robots Based on Decentralized Energy Pickup Structure," *IEEE Transactions on Industrial Informatics*, vol. 14, no. 4, pp. 1786-1797, 2018.

[6] A. Sample, D. J. Yeager, P. S. Powledge, A. V. Mamishev, and J. R. Smith, "Design of an RFID-based Battery-Free Programmable Sensing Platform," *IEEE Transactions on Instrumentation and Measurement*. IEEE, vol. 57, no.11, pp. 2608-2615, 2008.

[7] S. Wu, H. Dai, L. Xu, L. Liu, F. Xiao and J. Xu, "Comprehensive Cost Optimization for Charger Deployment in Multi-hop Wireless Charging," *IEEE Transactions on Mobile Computing*, 2022. doi: 10.1109/TMC.2022.3162112.

[8] C. Lin, Y. Zhou, F. Ma, J. Deng, L. Wang and G. Wu, "Minimizing Charging Delay for Directional Charging in Wireless Rechargeable Sensor Networks," in *IEEE INFOCOM*. IEEE, 2019, pp. 1819-1827.

[9] T. Wu, P. Yang and H. Dai, "Charging on the Move: Scheduling Static Chargers with Tunable Power for Mobile Devices," in *IEEE/ACM International Symposium on Quality of Service (IWQOS)*. IEEE/ACM, 2021, pp. 1-10.

[10] J. Xu, S. Hu, S. Wu, K. Zhou, H. Dai and L. Xu, "Cooperative Charging as Service: Scheduling for Mobile Wireless Rechargeable Sensor Networks," in *IEEE ICDCS*. IEEE, 2021, pp. 685-695.

[11] G. Fan, Z. Yang, H. Jin, X. Gan and X. Wang, "Enabling Optimal Control under Demand Elasticity for Electric Vehicle Charging Systems," *IEEE Transactions on Mobile Computing*, vol. 21, no. 3, pp. 955-970, 2022.

[12] A. K. Gupta, S. Ghosh and M. R. Bhatnagar, "Pricing Scheme for UAV-Enabled Charging of Sensor Network," in *IEEE INDICON*. IEEE, 2021, pp. 1-6.

[13] R. M. Nauss, "An Improved Algorithm for the Capacitated Facility Location Problem", *Journal of the Operational Research Society*, vol. 29, no. 12, pp. 1195-1201, 1978.

[14] H. Dai, Y. Liu, N. Yu, C. Wu, G. Chen, T. He and Alex X. Liu, "Radiation Constrained Wireless Charger Placement," *IEEE/ACM Transactions on Networking*, vol. 29, no. 1, pp. 48-64, 2021.

[15] H. Dai, Y. Liu, G. Chen, X. Wu, T. He, A. X. Liu and H. Ma, "Safe Charging for Wireless Power Transfer," *IEEE/ACM Transactions on Networking*, vol. 25, no. 6, pp. 3531-3544, 2017.

[16] N. Wang, J. Wu and H. Dai, "Bundle Charging: Wireless Charging Energy Minimization in Dense Wireless Sensor Networks," in *IEEE ICDCS*. IEEE, 2019, pp.810-820.

[17] Y. Sun, C. Lin, H. Dai, P. Wang, L. Wang, G. Wu and Q. Zhang, "Trading off Charging and Sensing for Stochastic Events Monitoring in WRSNs," *IEEE/ACM Transactions on Networking*, vol. 30, no. 2, pp. 557–571, 2022.

[18] L. Liu, H. Zhang and J. Wu, "A Reciprocal Charging Mechanism for Electric Vehicular Networks in Charging-Station-Absent Zones," *IEEE Transactions on Mobile Computing*, 2021. doi: 10.1109/TMC.2021.3096328.

[19] P. Zhou, C. Wang and Y. Yang, "Static and Mobile Target k-Coverage in Wireless Rechargeable Sensor Networks," *IEEE Transactions on Mobile Computing*, vol. 18, no. 10, pp. 2430-2445, 2019.

[20] R. Jia, J. Wu, J. Lu, M. Li, F. Lin and Z. Zheng, "Energy Saving in Heterogeneous Wireless Rechargeable Sensor Networks," in *IEEE INFOCOM*. IEEE, 2022, pp. 1838-1847.

[21] R. Jia, J. Lu, J. Wu, X. Wang, Z. Zheng and M. Li, "Geometric Analysis of Energy Saving for Directional Charging in WRSNs," *IEEE Internet of Things Journal*, vol. 8, no. 6, pp. 4697-4710, 2021.

[22] W. Xu, W. Liang, X. Jia, H. Kan, Y. Xu and X. Zhang, "Minimizing the Maximum Charging Delay of Multiple Mobile Chargers Under the Multi-Node Energy Charging Scheme," *IEEE Transactions on Mobile Computing*, vol. 20, no. 5, pp. 1846-1861, 2021.

[23] C. Lin, F. Gao, H. Dai, J. Ren, L. Wang and G. Wu, "Maximizing Charging Utility with Obstacles through Fresnel Diffraction Model," in *IEEE INFOCOM*. IEEE, 2020, pp. 2046-2055.

[24] C. Lin, W. Yang, H. Dai, T. Li, Y. Wang, L. Wang, G. Wu and Q. Zhang, "Near Optimal Charging Schedule for 3-D Wireless Rechargeable Sensor Networks," *IEEE Transactions on Mobile Computing*, 2021. doi: 10.1109/TMC.2021.3137308.

[25] H. Dai, X. Wang, A. X. Liu, H. Ma, G. Chen and W. Dou, "Wireless Charger Placement for Directional Charging," *IEEE/ACM Transactions on Networking*, vol. 26, no. 4, pp. 1865-1878, 2018.

[26] C. Lin, F. Gao, H. Dai, L. Wang and G. Wu, "When Wireless Charging Meets Fresnel Zones: Even Obstacles Can Enhance Charging Efficiency," in *IEEE SECON*. IEEE, 2019, pp. 1-9.

[27] K. Jung, J. Park, S. Son and S. Ahn, "Position Prediction of Wireless Charging Electric Vehicle for Auto Parking using Extreme Gradient Boost Algorithm," in *IEEE Wireless Power Transfer Conference (WPTC)*. IEEE, 2020, pp. 439-442.

[28] P. K. Chittoor and C. Bharatiraja, "Wireless-Sensor Communication Based Wireless-Charging Coil Positioning System for UAVs With Maximum Power Point Tracking," *IEEE Sensors Journal*, vol. 22, no. 8, pp. 8175-8182, 2022.

[29] I. Cortes and W. Kim, "Autonomous Positioning of a Mobile Robot for Wireless Charging Using Computer Vision and Misalignment-Sensing Coils," in *IEEE Annual American Control Conference (ACC)*. IEEE, 2018, pp. 4324-4329.

[30] A. Adeel, M. Gogate, S. Farooq, C. Ieracitano, K. Dashtipour, H. Larijani and A. Hussain, "A Survey on the Role of Wireless Sensor Networks and IoT in Disaster Management," *Geological Disaster Monitoring based on Sensor Networks*. Springer, pp. 57-66, 2019.

[31] X. Fan, L. Shangguan, R. Howard, Y. Zhang, Y. Peng, J. Xiong, Y. Ma and X. Li, "Towards Flexible Wireless Charging for Medical Implants Using Distributed Antenna System," in *ACM MobiCom*. ACM, 2020, pp. 1–15.

[32] A. Azmoodeh, A. Dehghantanha and K. R. Choo, "Robust Malware Detection for Internet of (Battlefield) Things Devices Using Deep Eigenspace Learning," *IEEE Transactions on Sustainable Computing*, vol. 4, no. 1, pp. 88-95, 2019.

[33] Y. Chi, Y. Li, J. Ye, L. Li, K. Xu and X. Liang, "Study on Requirements for Test and Detection of Substation Inspection Robot," in *International Conference on Energy, Electrical and Power Engineering (CEEPE)*. IEEE, 2022, pp. 1062-1067.

[34] X. Han and C. Liu, "Design of Autonomous Navigation Algorithm For Security Inspection Robot," in *International Conference on Computer, Network, Communication and Information Systems (CNCI)*. IEEE, 2019, pp. 1-9.

[35] J. Li, T. Huang, S. Chen and Y. Yang, "Optimization Based on Taxi Carpooling Preferences and Pricing," in *International Conference on Natural Computation, Fuzzy Systems and Knowledge Discovery (ICNC-FSKD)*. IEEE, 2018, pp. 108-112.

[36] L. Jiang, P. Sun, J. Xu, D. Yang, L. Xu and Y. Shi, "Cooperative Package Assignment for Heterogeneous Express Stations," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 7, pp. 8467-8476, 2022.

[37] C. Wang, J. Li, F. Ye, and Y. Yang, "A Novel Framework of Multi-Hop Wireless Charging for Sensor Networks Using Resonant Repeaters," *IEEE Transactions on Mobile Computing*, vol. 16, no. 3, pp.617-633, 2017.

[38] *Powercast*. Accessed: 2022. [Online]. Available: https://www.powercastco.com.

Sixu Wu received the bachelor's degree in School of Computer Science from Nanjing University of Posts and Telecommunications, Jiangsu, China, in 2019. He is currently a doctoral student in the School of Computer Science at Nanjing University of Posts and Telecommunications. His main research interest is wireless rechargeable sensor network.

Haipeng Dai (M'14, SM'22) received the B.S. degree in the Department of Electronic Engineering from Shanghai Jiao Tong University, Shanghai, China, in 2010, and the Ph.D. degree in the Department of Computer Science and Technology in Nanjing University, Nanjing, China, in 2014. He is an associate professor in the Department of Computer Science and Technology in Nanjing University. He is an IEEE senior member and an ACM member. He received Best Paper Award from IEEE ICNP'15, Best Paper Award Runner-up from IEEE SECON'18, and Best Paper Award Candidate from IEEE INFOCOM'17.

Linfeng Liu (M'13) received the B. S. and Ph. D. degrees in computer science from the Southeast University, Nanjing, China, in 2003 and 2008, respectively. At present, he is a Professor in the School of Computer Science and Technology, Nanjing University of Posts and Telecommunications, China. His main research interests include the areas of vehicular ad hoc networks, wireless sensor networks and multi-hop mobile wireless networks. He has published more than 80 peer-reviewed papers in some technical journals or conference proceedings, such as IEEE TMC, IEEE TPDS, ACM TAAS, IEEE TSC, IEEE TVT, IEEE IoTJ, Computer Networks, Elsevier JPDC.

Lijie Xu received his Ph.D. degree in the Department of Computer Science and Technology from Nanjing University, Nanjing, in 2014. He was a research assistant in the Department of Computing at the Hong Kong Polytechnic University, Hong Kong, from 2011 to 2012. He is currently an associate professor in the School of Computer Science at Nanjing University of Posts and Telecommunications, Nanjing. His research interests are mainly in the areas of wireless sensor networks, ad-hoc networks, mobile and distributed computing, and graph theory algorithms.

Fu Xiao (M'12) received the Ph.D. degree in computer science and technology from the Nanjing University of Science and Technology, Nanjing, China, in 2007. He is currently a Professor in the Jiangsu Key Laboratory of Big Data Security and Intelligent Processing at Nanjing University of Posts and Telecommunications. His main research interests are wireless sensor networks and mobile computing. He is a member of the IEEE Computer Society and the Association for Computing Machinery.

Jia Xu (M'15, SM'21) received the M.S. degree in School of Information and Engineering from Yangzhou University, Jiangsu, China, in 2006 and the PhD. Degree in School of Computer Science and Engineering from Nanjing University of Science and Technology, Jiangsu, China, in 2010. He is currently a professor in the School of Computer Science at Nanjing University of Posts and Telecommunications. He was a visiting Scholar in the Department of Electrical Engineering & Computer Science at Colorado School of Mines from Nov. 2014 to May. 2015. His main research interests include crowdsourcing, edge computing and wireless sensor networks. Prof. Xu has served as the PC Co-Chair of SciSec 2019, Publicity Co-Chair of SciSec 2021 and SciSec 2022, Organizing Chair of ISKE 2017, TPC member of IEEE Globecom, IEEE ICC, IEEE MASS, IEEE ICNC, and IEEE EDGE.