# Incentive Mechanisms for Spatio-temporal Tasks in Mobile Crowdsensing\*

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Abstract—Mobile crowdsensing emerges as a new paradigm that takes advantage of pervasive sensor-embedded smartphones to collect sensory data. Many incentive mechanisms for mobile crowdsensing have been proposed. However, none of them takes into consideration the spatio-temporal tasks in mobile crowdsensing systems, where the sensing areas of tasks can have overlaps, and the collective sensing time for each task needs to meet the specified time duration. Motivated by the mobile crowdsensing system for pervasive traffic monitoring, we present two system models for location sensitive users and location insensitive users, respectively, and formulate the social optimization problem for each model. We design two reverse auction based truthful incentive mechanisms to minimize the social cost subject to the constraint that each of the spatio-temporal tasks can be completed with its collective sensing time no less than a minimum sensing time required by the platform. Through both theoretical analysis and extensive simulations, we demonstrate that the proposed mechanisms achieve computational efficiency, individual rationality, truthfulness, and guaranteed approximation.

*Index Terms*—Mobile crowdsensing, Incentive mechanism, Reverse auction, Spatio-temporal tasks

# I. INTRODUCTION

In the past few years, the market of smartphone has proliferated rapidly and continues to expand. According to IDC, worldwide smartphone shipments are expected to reach 1.42 billion units in 2018. Over the long-term, smartphone shipments are forecast to reach 1.57 billion units in 2022 [1]. With arise of the 4G/5G networks and the embedded sensors, such as accelerometer, digital compass, gyroscope,

This work has been supported in part by the NSFC (No. 61872193), and NSF (No. 1717315).

GPS, and camera, the smartphone gradually becomes a powerful programmable mobile data interface. These sensors can sense various human activities and surrounding environment cooperatively. Mobile crowdsensing outsources the collection of sensory data to a crowd of participating users, who usually carry increasingly capable mobile devices (e.g., smartphones, smartwatches, and smartglasses) with a plethora of on-board sensors. Generally, mobile crowdsensing has become an efficient approach to meeting the demands in large-scale sensing applications, such as trip selection for public bike system [2], real-time traffic management system [3], and SmartBike [4] for monitoring city air pollution.

Incentive mechanisms are crucial to mobile crowdsensing since the smartphone users spend their time and consume battery, memory, computing power and data traffic of device to sense, store and transmit the data. Moreover, there are potential privacy threats to smartphone users by sharing their sensed data with location tags, interests or identities. A lot of efforts [5-7] have been focused on developing incentive mechanisms to entice users to participate in crowdsensing. Generally, the requirements of time and/or location of crowdsensing tasks have great influence on the task allocation. Some incentive mechanisms [8-10] consider the crowdsensing system, where the tasks are location dependent, i.e., the tasks only can be performed by the users who are in the specific sensing positions or areas. The incentive mechanisms for time dependent crowdsensing tasks [11-13] consider that the crowdsensing tasks are with time property, i.e., each task requires the sensory data of time duration.

In many crowdsensing scenarios (e.g., environment monitoring, traffic monitoring), users are required to sense in specific areas for sufficient time. The task in above scenarios falls into the range of general spatio-temporal task, which requires the sensing data with sufficient time from sufficient sensing areas. In some scenarios, the platform requires the sensing data within a specific time window [12, 13]. There may exist both spatial correlation and time correlation. For example, both the sensing areas and the time window of tasks may have overlaps.

The mobile crowdsensing scenarios with such spatiotemporal tasks are very pervasive [9-12]. Taking traffic monitoring illustrated in Figure 1 as an example, the mobile crowdsensing platform requires traffic videos reported by users for all roads, and the traffic videos should be sufficient for the purpose of data integration, analysis, or prediction. Thus, the collective traffic videos for each road need to meet a requirement of specific time duration. The users can move along the road and record traffic video in any position of the road. Note that a user in the crossroad can record traffic video for every road passed through the crossroad simultaneously. In this traffic monitoring scenario, the roads can be seen as the *AoIs* (*Area of Interests*) that overlap with each other at the crossroads. Obviously, collection of the traffic videos for each road is a spatio-temporal task.

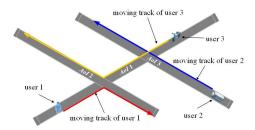


Fig. 1. Motivating example of spatio-temporal tasks in mobile crowdsensing.

It is important to consider the overlapping of sensing areas. First, the tasks may be publicized by different requesters, and the overlapping of sensing areas is a common phenomenon in mobile crowdsensing. More importantly, the data reuse in the overlapped sensing areas can reduce the social cost of users.

However, there is no off-the-shelf incentive mechanism designed for the special case of spatio-temporal tasks in mobile crowdsensing systems, where the sensing areas of tasks can have overlaps, and the collective sensing time for each task needs to meet the specified time duration. In this paper, we aim to design truthful incentive mechanism for such spatio-temporal tasks in mobile crowdsensing. Consider that the platform announces a set of sensing tasks with sensing areas that may overlap with each other and minimum sensing time required for each. The users can move in their own active areas. The objective of our incentive mechanism is to minimize the social cost subject to the constraint that each task can be completed with its sensing time no less than a given minimum sensing time.

We model the mobile crowdsensing process as a sealed reverse auction. In our system model, each user can bid with an active area he/she can move in and a maximum sensing time he/she can spend in performing tasks. The platform selects a subset of users and notifies the selected winners, and allocates the sensing time of each winner for each task. The winners perform the tasks in their active areas according to the allocated sensing time. Finally, each winner obtains the payment, which is determined by the platform. The whole process is illustrated by Figure 2.



Fig. 2. Reverse auction framework for mobile crowdsensing with spatiotemporal tasks.

The problem of designing truthful incentive mechanisms to minimize the social cost for spatio-temporal tasks in mobile crowdsensing is very challenging. First, the sensing areas of tasks have overlaps, and the users located in the overlaps can contribute to multiple tasks simultaneously. We need to determine the value of sensing data provided by such users. Second, different from most crowdsesing models, the users can move to any position, which may belong to different sensing areas of tasks, within their active areas. We need an efficient method to allocate the sensing time of the users for their sensing areas rather than simply allocate the tasks to the users. Moreover, each user may take a strategic behavior by submitting dishonest bidding price to maximize its utility.

The main contributions of this paper are as follows:

- To the best of our knowledge, this is the first work to design incentive mechanisms for spatio-temporal tasks in mobile crowdsensing systems, where the AoIs of tasks can have overlaps, and the collective sensing time for each task need to meet the specific requirement of time duration.
- We present two reverse auction based system models: location sensitive model and location insensitive model, and formulate the social optimization problems for both models.
- We present a sensing time allocation algorithm to allocate the sensing time of each user for each task.
- We design an incentive mechanism for each of two models. We show that the designed mechanisms satisfy desirable properties of computational efficiency, individual rationality, truthfulness, and guaranteed approximation.

The rest of the paper is organized as follows. Section 2 formulates the system models and problems, and lists some desirable properties. Section 3 and Section 4 present the detailed design and analysis of our incentive mechanisms for both two system models, respectively. Performance evaluation is presented in Section 5. We review the state-of-art research in Section 6, and conclude this paper in Section 7.

## II. SYSTEM MODEL AND DESIRABLE PROPERTIES

In this section, we model the mobile crowd sensing system as a reverse auction and present two different models: location sensitive model and location insensitive model. In the location sensitive model, each user is associated with a specific geographic position (such as office and residence), and can perform the sensing tasks whose *AoIs* cover the position. The *AoI* of a task is an area, where the platform is interested in collecting the sensing data. In location insensitive model, each user has an active area, and the user can move to any position in this active area to perform sensing tasks, of which the *AoIs* overlap the active area. In reality, the active areas of users can be the areas the users live in. For example, a college student's campus can be viewed as one of his/her active area. At the end of this section, we present some desirable properties.

## A. Location Sensitive Model

We consider a mobile crowdsensing system consisting of a platform and many smartphone users. The platform resides in a cloud. The smartphone users connect the platform via the cloud. The platform publicizes a set  $\Gamma = \{\tau_1, \tau_2, ..., \tau_m\}$  of m homogeneous sensing tasks, and seeks to collect the sensory data with same type, such as air pollution readings [14] or noise levels [15] in multiple places in a city. Each task  $\tau_j, j = 1, 2, ..., m$ , is associated with a pair of spatiotemporal property  $(t^j, a^j)$ , where  $t^j$  is the minimum sensing time required to complete the task, and  $a^j$  is the associated AoI of this task. Without loss of generality, the AoIs can be of any shape. For convenience, we consider that the sensing time is discrete, and the sensing time mentioned in this paper can be viewed as the number of time units.

Assume that a crowd  $U = \{1, 2, ..., n\}$  of n smartphone users are interested in participating sensing tasks. Each user  $i \in U$  submits a bid  $B_i = (t_i, l_i, b_i)$  to the platform, where  $t_i$  is the maximum sensing time the user i can spend in performing the tasks,  $l_i$  is the position of the user i, and  $b_i$ is user i's bidding price for spending time of  $t_i$  to perform the tasks. Each user i also has a cost  $c_i$ . We consider that  $c_i$  is the private information and is known only to user i. Different from most mobile crowdsensing systems [16-18], in our system model, the users only bid for the sensing time rather than specific tasks. Let  $U^{j}$  be the set of users whose positions are in the AoI of any task  $\tau_j$ , j = 1, 2, ..., m. Let  $\Gamma_i = \{\tau_j \mid l_i \in a^j, \forall \tau_j \in \Gamma\}$  be the set of tasks, of which the AoIs cover the position of user i. In reality, the overlaps of *AoIs* for the given position are small. Thus we consider  $|\Gamma_i|$ is bounded by  $\varepsilon$ , i.e.,  $\max_{i \in U} |\Gamma_i| \leq \varepsilon$ , where  $\varepsilon$  is a small positive integer.

Note that the *AoIs* of tasks may have overlaps, and all tasks are homogeneous. Thus a user located in the overlaps can sense for multiple tasks simultaneously. As illustrated in Figure 3, the user 2 can sense for all three tasks simultaneously because the *AoIs* of these three tasks cover the position of user 2. Accordingly, user 3 can sense for two tasks simultaneously, and user 1 only can sense for one task.

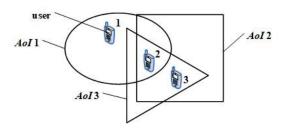


Fig. 3. An example of location sensitive model. There are three users and three tasks with *AoIs*.

Given the task set  $\Gamma$  and the bid profile  $\mathbf{B} = (B_1, B_2, ..., B_n)$ , the platform calculates the winning set  $S \subseteq U$ , and notifies winners of the determination. The winners perform the sensing tasks in their positions and send data back to the platform. Each user i is paid  $p_i$  by the platform.

We define the utility of user i as the difference between the payment and its real cost:

$$u(i) = \begin{cases} p_i - c_i & \text{if } i \in S \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Specially, the utility of the losers would be zero because they are paid nothing in our designed mechanisms and there is no cost for sensing.

Note that  $b_i$  can be different from the real cost  $c_i$  because we consider the users selfish. So, the users may take a strategic behavior by claiming dishonest cost to maximize their own utility. We consider that the maximum sensing time and positions reported by the users are always truthful since the platform can check whether the assigned tasks are performed with declared sensing time.

The monopoly is harmful to crowdsensing system since the monopolist can manipulate the price and quality of sensing data. To prevent monopoly, we assume that all spatio-temporal tasks still can be completed if any user does not participate in the auction. This assumption is reasonable for crowdsensing systems as shown in [8, 12, 13].

Next, we define the utility of the platform as:

$$u_0 = V\left(\Gamma\right) - \sum_{i \in S} p_i \tag{2}$$

where  $V(\Gamma)$  is the value to the platform when it obtains all sensory data with sensing time no less than the minimum sensing time for every task in  $\Gamma$ .

Moreover, the social welfare is:

$$u_{soc} = u_0 + \sum_{i \in S} u_i = V(\Gamma) - \sum_{i \in S} c_i$$
 (3)

The incentive mechanism  $\mathcal{M}(\Gamma, \mathbf{B})$  outputs a winning set S and a payment profile  $\mathbf{p} = (p_1, p_2, ..., p_n)$ . The objective is maximizing the social welfare subject to the constraint that each of the tasks in  $\Gamma$  can be completed with the collective sensing time no less than the minimum sensing time.

The problem of maximizing social efficiency is equivalent to the problem of minimizing the social cost since all tasks in  $\Gamma$  are required to be completed according to Equation (3). We call this problem *Location Sensitive Social Optimization* (*LSSO*) problem, which can be formulated as follows:

Objective: 
$$\min \sum_{i \in S} c_i$$
 (4)

Subject to: 
$$\sum_{i \in U^j} t_i \ge t^j, \forall \tau_j \in \Gamma$$
 (5)

# B. Location Insensitive Model

In this subsection, we consider the case where each user is location insensitive. Particularly, each user  $i \in U$  submits a bid  $B_i = (t_i, a_i, b_i)$  to the platform, where the definitions of  $t_i$  and  $b_i$  are same as those in location sensitive model.  $a_i$  is the active area of user i, which also can be of any shape. The active areas can be determined through the future schedules or daily mobility routines with little effect on their daily life. Different from the location sensitive model, the users are willing to move to any position within the active areas to perform the tasks, of which the AoIs overlap its active area.

Another key factor for determining the active area is the number of tasks the user is willing to perform. Let  $\Gamma_i = \{\tau_j | a_i \cap a^j \neq \emptyset, \forall \tau_j \in \Gamma\}$  be the set of tasks whose AoIs overlap the active area of any user i. In reality, the active area can be a road the user passes through, or the area of the user's company. Thus we also consider  $|\Gamma_i|$  is bounded by  $\varepsilon$ .

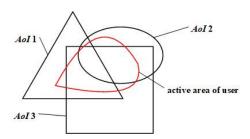


Fig. 4. An example of location insensitive model. There are one user with active area and three tasks with *AoIs*.

Similar to the location sensitive model, a user located in the overlaps of *AoIs* can sense for multiple tasks simultaneously. As illustrated in Figure 4, there are seven potential sensing areas within the active area of the user. Thus the specific problem in location insensitive model is how to allocate the time spend on multiple sensing area in the active area for each winner. Since the sensing areas can exist with any overlap of the member of power set of  $|\Gamma_i|$  except the empty set, there are at most  $2^{|\Gamma_i|} - 1$  sensing areas for any user i. Let  $SA_i = \{sa_{i,1}, sa_{i,2}, ..., sa_{i,2|\Gamma_i|-1}\}$  be the set of sensing areas of user i. Let  $|sa_{i,k}|, k = 1, 2, ..., 2^{|\Gamma_i|} - 1$ , represent the number of tasks overlapped in sensing area  $sa_{i,k}$ . Let  $tsa_{i,k}$  be the sensing time of user i allocated in sensing area  $sa_{i,k}$ . Let **TSA** be the sensing time allocation for all users. To maximize the value provided by each user, the platform will always prefer allocating the sensing time to the sensing area with maximum overlaps.

We also consider that the users may take a strategic behavior by claiming dishonest cost to maximize their own utility. The

TABLE I FREQUENTLY USED NOTATIONS

Γ	task set	$ au_j$	task j				
$t^j$	minimum sensing time of task $j$	$a^j$	AoI of task $j$				
U	user set	$B_i$	bid of user i				
$t_i$	maximum sensing time of user $i$	$l_i$	position of user i				
$b_i$	bidding price of user $i$	$c_i$	cost of user i				
$a_i$	active area of user i	$SA_i$	set of sensing areas of user i				
TSA	sensing time allocation	$sa_{i,k}$	kth sensing area of user i				
ε	maximum size of	В	bid profile				
$p_i$	payment to user i	р	payment profile				
S	winner set	$u_{soc}$	social welfare				
$u_i$	utility of user $i$	$u_0$	utility of the platform				
$U^{j}$	set of users whose positions are in the $AoI$ of task $j$						
$\Gamma_i$	set of tasks whose $AoIs$ cover/overlap the position/sensing area of user $i$						
$V\left(\Gamma\right)$	value to the platform when all tasks in $\Gamma$ are performed						
$tsa_{i,k}$	sensing time of user $i$ allocated in sensing area $sa_{i,k}$						

incentive mechanism  $\mathcal{M}(\Gamma, \mathbf{B})$  outputs a winning set S, the sensing time allocation **TSA**, and a payment profile  $\mathbf{p}$ . We formulate the *Location Insensitive Social Optimization (LISO)* problem as follows:

Objective: 
$$\min \sum_{i \in S} c_i$$
 (6)

Subject to: 
$$\sum\nolimits_{i \in U, sa_{i,k} \cap a^{j} \neq \emptyset} tsa_{i,k} \geq t^{j}, \forall \tau_{j} \in \Gamma \quad (7)$$

The definition of *LISO* problem shows the objective of selecting the winners with minimum social cost, which is the sum of the real costs of winners. The constraint shows that the platform guarantees that each sensing task can be finished, i.e., the sum of sensing time allocated in the sensing areas, which overlap the *AoI* of any task is no less than the minimum sensing time of this task required by the platform.

*Remark:* The hybrid scenario with both location sensitive users and location insensitive users can be viewed as the special case of location insensitive model by setting the active areas of location sensitive users to the points.

# C. Desirable Properties

Our objective is to design the incentive mechanisms satisfying the following desirable properties:

- Computational Efficiency: An incentive mechanism is computationally efficient if the winner set S, (sensing time allocation TSA if necessary) and the payment p can be computed in polynomial time.
- Individual Rationality: Each user will have a non-negative utility while reporting true private information, i.e.,  $u_i \geq 0, \forall i \in U$ .
- **Truthfulness:** A mechanism is truthful if no user can improve its utility by submitting false cost, no matter

what others submit. In other words, reporting the real cost is a weakly dominant strategy for all users.

 Approximation Ratio: The goal of the mechanism is to minimize the social cost. We attempt to find the algorithms with low approximation ratios.

The importance of the first two properties is obvious, because they together ensure the feasibility of the incentive mechanism. The last two properties are indispensable for guaranteeing the compatibility and high performance. Being truthful, the incentive mechanism can eliminate the fear of market manipulation and the overhead of strategizing over others for the participating users.

We list the frequently used notations in Table 1.

# III. INCENTIVE MECHANISM FOR LOCATION SENSITIVE USERS

In this section, we present an *Incentive Mechanism in Location Sensitive Model (MLS)*.

# A. Mechanism Design

First of all, we attempt to find an efficient algorithm for the *LSSO* problem presented in Equation (4) and (5). Unfortunately, as the following theorem shows, it is NP-hard to find the optimal solution.

# **Theorem 1.** The LSSO problem is NP-hard.

Proof. We consider a corresponding instance of LSSO: Let  $\Gamma = \{\tau_1, \tau_2, ..., \tau_m\}$  be the universe set of all tasks publicized by the platform. Each task  $\tau_j, j \in \{1, 2, ..., m\}$  has a minimum sensing time  $t^j$ . For a family of task multi-set  $T = \{T_1, T_2, ..., T_n\}$ , each user i is associated with a task multi-set  $T_i$  and a cost  $c_i$ , where  $T_i$  includes all the tasks in  $\Gamma_i$  defined in Subsection 2.1, and each task  $\tau_j \in \Gamma_i$  is with a multiplicity  $\min\{t_i, t^j\}$ . The question is whether there exists a set  $T' \subseteq T$  with  $\sum_{T_i \in T'} c_i < v$ , such that each element  $\tau_j$  can be covered at least  $t^j$  times. Then we can see that this problem is the Multi-set Multi-cover (MSMC) problem. Since the MSMC problem is a well-known NP-hard problem, the LSSO problem is NP-hard.

Since the *LSSO* problem is NP-hard, it is impossible to compute the winner set with minimum social cost in polynomial time unless P=NP. In addition, we cannot use the off-the-shelf VCG mechanism [19] since the truthfulness of VCG mechanism requires that the social cost is exactly minimized. Our auction based incentive mechanism follows a greedy approach. As illustrated in Algorithm 1, *MLS* consists of winner selection phase and payment determination phase.

In the winner selection phase, the users are sorted according to the effective average cost. Given the uncovered time units of each task  $t^{'j}$ ,  $\tau_j$  in  $\Gamma_i$ , the effective coverage of user i is  $\min\{t_i,t^{'j}\}$ . The effective average cost of user i is defined as  $\frac{b_i}{\sum_{\tau_j\in\Gamma_i}\min\{t_i,t^{'j}\}}$ . In each iteration of the winner selection phase, we select the user with minimum effective average cost over the unselected user set  $U\backslash S$  as the winner until the

# Algorithm 1 : MLS

```
Input: task set \Gamma, user set U, bid profile B
Output: winner set S, payment profile p
         //Phase 1: Winner Selection
   1: S \leftarrow \varnothing:
  2: for all \tau_i \in \Gamma do
          t^{'j} \leftarrow t^j;
  4: end for
  5: while \sum_{\tau_j \in \Gamma} t'^j \neq 0 do
6: i \leftarrow \arg \min_{h \in U \setminus S} \frac{b_h}{\sum_{\tau_j \in \Gamma_h} \min\{t_h, t'^j\}};
               S \leftarrow S \cup \{i\};
              for all \tau_j \in \Gamma_i do t'^j \leftarrow t'^j - \min\{t_i, t'^j\};
  9:
 10:
 11: end while
         //Phase 2: Payment Determination
 12: for all i \in U do
               p_i \leftarrow 0;
 14: end for
 15: for all i \in S do
              \begin{aligned} & u' \leftarrow U \setminus \{i\}, S' \leftarrow \emptyset, t''^j \leftarrow t^j; \\ & \mathbf{while} \ \sum_{\tau_j \in \Gamma} t''^j \neq 0 \ \mathbf{do} \\ & i_h \leftarrow \arg \min_{h \in U' \setminus S'} \frac{b_h}{\sum_{\tau_j \in \Gamma_h} \min\{t_h, t''^j\}}; \end{aligned}
 18:
                   S^{'} \leftarrow S^{'} \cup \{i_{h}\};
p_{i} \leftarrow \max\{p_{i}, \frac{\sum_{\tau_{j} \in \Gamma_{i}} \min\{t_{i}, t^{''j}\}}{\sum_{\tau_{j} \in \Gamma_{i_{h}}} \min\{t_{i_{h}}, t^{''j}\}} b_{i_{h}}\};
\mathbf{for all } \tau_{j} \in \Gamma_{i_{h}} \mathbf{do}
20:
21:
                          t''^{j} \leftarrow t''^{j} - \min\{t_{i_h}, t''^{j}\};
22:
                     end for
23:
24:
               end while
25: end for
```

winners' sensing time can meet the requirement of minimum sensing time of each task in  $\Gamma$ .

In payment determination phase, for each winner  $i \in S$ , we execute the winner selection phase over  $U \setminus \{i\}$ , and the winner set is denoted by S'. We compute the maximum price that user i can be selected instead of each user in S'. We will prove that this price is a critical payment for user i later.

## B. Mechanism Analysis

In the following, we present theoretical analysis, demonstrating that *MLS* can achieve the desired properties of computational efficiency, individual rationality, truthfulness and low approximation ratio.

# **Lemma 1.** MLS is computationally efficient.

*Proof.* Finding the user with minimum effective average cost takes  $O(n\varepsilon)$ , where computing the value of  $\sum_{\tau_j \in \Gamma_h} \min\{t_h, t^{'j}\}$  takes  $O(\varepsilon)$  time. Hence, the while-loop (Lines 5-11) takes  $O(n^2\varepsilon)$ . In each iteration of the for-loop (Lines 15–25), a process similar to line 5–11 is executed. Hence the running time of the whole auction is dominated by this for-loop, which is bounded by  $O(n^3\varepsilon)$ .

# **Lemma 2.** MLS is individually rational.

*Proof.* Let  $i_h$  be user i's replacement which appears in the *i*th place in the sorting over  $U \setminus \{i\}$ . Since user  $i_h$ would not be at the ith place if i is considered, we have where the tent place if the is considered, we have  $\frac{b_i}{\sum_{\tau_j \in \Gamma_i} \min\{t_i, t'^j\}} \leq \frac{b_{i_h}}{\sum_{\tau_j \in \Gamma_{i_h}} \min\{t_{i_h}, t'^j\}}. \text{ Hence we have } b_i \leq \frac{\sum_{\tau_j \in \Gamma_i} \min\{t_i, t'^j\}}{\sum_{\tau_j \in \Gamma_{i_h}} \min\{t_i, t'^j\}} b_{i_h} = \frac{\sum_{\tau_j \in \Gamma_i} \min\{t_i, t'^j\}}{\sum_{\tau_j \in \Gamma_{i_h}} \min\{t_i, t'^j\}} b_{i_h}, \text{ where the } b_i = \frac{\sum_{\tau_j \in \Gamma_i} \min\{t_i, t'^j\}}{\sum_{\tau_j \in \Gamma_{i_h}} \min\{t_i, t'^j\}} b_{i_h}$ equality relies on the observation that  $t'^{j} = t''^{j}$  for every  $h \le$ i, which is due to the fact that S = S' for every  $h \le i$ . This is sufficient to get  $b_i \leq \max_{h \in U' \setminus S'} \frac{\sum_{\tau_j \in \Gamma_i} \min\{t_i, t^{''j}\}}{\sum_{\tau_j \in \Gamma_{i_h}} \min\{t_{i_h}, t^{''j}\}} b_{i_h} =$  $p_i$ .

Before analyzing the truthfulness of MLS, we firstly introduce the Myerson's Theorem [18].

**Theorem 2** (5, Theorem 2.1). : An auction mechanism is truthful if and only if:

- The selection rule is monotone: If user i wins the auction by bidding  $b_i$ , it also wins by bidding  $b_i \leq b_i$ ;
- Each winner is paid the critical value: User i would not win the auction if it bids higher than this value.

Proof. Based on Theorem 2, it suffices to prove that the

# Lemma 3. MLS is truthful.

selection rule of MLS is monotone and the payment  $p_i$  for each i is the critical value. The monotonicity of the selection rule is obvious as bidding a smaller value cannot push user i backwards in the sorting. We next show that  $p_i$  is the critical value for i in the sense that bidding higher than  $p_i$ could prevent i from winning the auction. Note that  $p_i$  $\max_{h \in \{1,...,L\}} \frac{\sum_{\tau_j \in \Gamma_i} \min\{t_i, t''^j\}}{\sum_{\tau_j \in \Gamma_{i_h}} \min\{t_{i_h}, t''^j\}} b_{i_h}$ . If user i bids  $b_i \geq p_i$ , it will be placed after L since  $b_i \geq \frac{\sum_{\tau_j \in \Gamma_i} \min\{t_i, t^{''j}\}}{\sum_{\tau_j \in \Gamma_{i_L}} \min\{t_{i_L}, t^{''j}\}} b_{i_L}$ implies  $\frac{b_i}{\sum_{\tau_j \in \Gamma_i} \min\{t_i, t'^j\}} \ge \frac{b_{i_L}}{\sum_{\tau_j \in \Gamma_{i_L}} \min\{t_{i_L}, t'^j\}}$ . Hence, user i would not win the second ii would not win the auction because the first L users have met the requirement of minimum sensing time of each task in Γ. 

**Lemma 4.** MLS can approximate the optimal solution within a factor of  $H_K$ , where  $K = \max_{i \in U} \sum_{\tau_i \in \Gamma_i} \min\{t_i, t^j\}, H_K =$  $1 + \frac{1}{2} + \dots + \frac{1}{K}$ .

Proof. We formulate the linear program relaxation of the LSSO problem defined in Equation (4) and (5) as the normalized primal linear program LP. The dual program is formulated in program DP.

$$\mathbf{LP:} \qquad \min \sum_{i \in II} c_i x_i \tag{8}$$

**s.t.** 
$$\sum_{i \in U} m(T_i, \tau_j) x_i \ge t^j, \forall \tau_j \in \Gamma$$
 (9) 
$$0 < x_i < 1, \forall i \in U$$
 (10)

$$\mathbf{DP}: \max \sum_{\tau_i \in \Gamma} t^j y_j - \sum_{i \in U} z_i \tag{11}$$

**s.t.** 
$$\sum\nolimits_{i \in \Gamma} m\left(T_{i}, \tau_{j}\right) y_{j} - z_{i} \leq c_{i}, \forall i \in U \quad (12)$$

$$z_i \ge 0, \forall i \in U \tag{13}$$

$$y_j \ge 0, \forall \tau_j \in \Gamma \tag{14}$$

where  $m(i, \tau_i) = \min\{t_i, t^j\}, \tau_i \in \Gamma_i$ , and  $T_i$  is a multi-set of tasks defined in the proof of Theorem 1.

When user i is selected, its cost  $c_i$  is ascribed equally to each tuple  $(\tau_j, e)$  where  $T_i$  covers  $\tau_j$  for the eth time,  $\forall e \in \{1, 2, ..., t^j\}$ . We say  $T_i$  covers  $(\tau_i, e)$  in short. Let  $cost(\tau_i, e)$  be the effective average cost defined in Subsection 3.1 when  $T_i$  covers  $(\tau_j, e)$ . Let  $\alpha_j = cost(\tau_j, t^j)$ , and  $\beta_i = \sum_{(\tau_j, e) is \ covered \ by \ T_i} \left( cost(\tau_j, t^j) - cost(\tau_j, e) \right).$ Obviously, both  $\alpha_j$  and  $\beta_i$  are nonnegative since  $cost(\tau_j, t^j)$  –  $cost(\tau_i, e) \ge 0$  based on MLS, for  $\forall e \in \{1, 2, ..., t^j - 1\}$ . If i is not selected,  $\beta_i$  is defined to be 0. Moreover, the objective function value of the dual solution  $(\alpha, \beta)$  is  $\sum_{\tau_j \in \Gamma} t_j \alpha_j$  $\sum_{i \in U} \beta_i = \sum_{\tau_j \in \Gamma} \sum_{e=1}^{t^j} cost(\tau_j, e)$ , which is the total cost produced by *MLS*.

Now let  $y_j = \frac{1}{H_K} cost(\tau_j, t^j)$ , where  $K = max_{i \in U} |T_i|$  =  $\max_{i \in U} \sum_{\tau_j \in \Gamma_i} \min\{t_i, t^j\}$  is the maximum size of all multi-sets.  $H_K$  is the harmonic number of K. If iis not selected, let  $z_i = 0$ , and otherwise let  $z_i =$  $\begin{array}{l} \frac{1}{H_{K}}\sum_{(\tau_{j},e)is\ covered\ by\ T_{i}}\left(cost\left(\tau_{j},t^{j}\right)-cost\left(\tau_{j},e\right)\right). \\ \text{We then show that all pairs }\left(y_{j},z_{i}\right),\tau_{j}\in\Gamma,i\in U \text{ are } \end{array}$ 

feasible to the dual program **DP**. Obviously, both  $y_i$  and  $z_i$ are nonnegative since  $cost(\tau_i, t^j) - cost(\tau_i, e) \ge 0$ , for  $\forall e \in$  $\{1, 2, ..., t^j - 1\}$  based on MLS. Consider for any  $T_i \in T$ , we

$$\sum_{\tau_{j} \in \Gamma} m\left(T_{i}, \tau_{j}\right) y_{j} - z_{i}$$

$$= \frac{1}{H_{K}} \left(\sum_{\tau_{j} \in \Gamma} m\left(T_{i}, \tau_{j}\right)\right) cost\left(\tau_{j}, t^{j}\right)$$

$$-\frac{1}{H_{K}} \sum_{(\tau_{j}, e) \ covered \ by \ T_{i}} \left(cost\left(\tau_{j}, t^{j}\right) - cost\left(\tau_{j}, e\right)\right)$$

$$= \frac{1}{H_{K}} \sum_{(\tau_{j}, e) \ covered \ by \ T_{i}} cost\left(\tau_{j}, e\right)$$

$$+\frac{1}{H_{K}} \sum_{\tau_{j} \in T_{i}, \ not \ covered \ by \ T_{i}} cost\left(\tau_{j}, t^{j}\right)$$

We sort the tasks in  $T_i$  according to the reverse order in which they were covered fully. Then the effective average cost of the task in place j in this order is at most  $c_i/j$ . Thus, we have  $\sum_{(\tau_j \in \Gamma)} m(T_i, \tau_j) y_j - z_i \leq \frac{1}{H_K} \left(\sum_{j=1}^{K} \frac{1}{j}\right) c_i = c_i$ . Hence, the pairs  $(y_j, z_i), \tau_j \in \Gamma, i \in U$  are feasible to the dual program DP. As a consequence, the total cost produced by MLS is  $\sum_{\tau_i \in \Gamma} t^j \alpha_j - \sum_{i \in U} \beta_i =$  $H_K\left(\sum_{\tau_j\in\Gamma}t^jy_j-\sum_{i\in U}z_i\right)\leq H_K\cdot OPT$  This completes the proof.

The above four lemmas together prove the following theorem.

**Theorem 3.** MLS is computationally efficient, individually rational, truthful, and  $H_K$  approximate, where  $K = \max_{i \in U} \sum_{\tau_j \in \Gamma_i} \min\{t_i, t^j\}$ .

# IV. INCENTIVE MECHANISM FOR LOCATION INSENSITIVE USERS

In this section, we consider the case, where the users are location insensitive, and present an *Incentive Mechanism in Location Insensitive Model (MLI)*.

# A. Mechanism Design

First of all, as the following theorem shows, the *LISO* problem presented in Equation (6) and (7) is also NP-hard to find the optimal solution.

# **Theorem 4.** The LISO problem is NP-hard.

*Proof.* We show that the *LISO* problem is equivalent to the *MSMC* problem. The proof is similar to Theorem 1. The difference is the setting of the family of multi-set  $T=\{T_1,T_2,...,T_n\}$  in the instance of *LISO*. Let each  $T_i\in T$  includes all tasks in  $\Gamma_i$  defined in Subsection 2.2. Since the platform will always prefer allocating the sensing time to the sensing area with maximum overlaps, the sensing time for each sensing area can be determined. Then we set  $\sum_k tsa_{i,k}, \forall sa_{i,k} \in SA_i, sa_{i,k} \cap a^j \neq \emptyset$  as the multiplicity of each task  $\tau_i \in \Gamma_i$ .

*Remark:* Since the *LISO* problem is a generalization of the *LSSO* problem, it suffices to prove that the *LISO* problem is NP-hard. Here, we show the equivalence between the *LISO* problem and the *MSMC* problem to assist the approximation analysis of *MLI*, which will be given in Theorem 5.

As illustrated in Algorithm 2, *MLI* also consists of winner selection phase and payment determination phase.

The winner selection phase also follows the greedy approach. The users are sorted according to the effective average cost. Given the uncovered time units of each task  $t'^j, \tau_j \in \Gamma_i$ , the effective coverage of user i is  $\sum_{k:sa_{i,k}\cap a^j\neq\emptyset} tsa_{i,k}$ , i.e., the total sensing time allocated in the sensing areas, which overlap the AoI of task  $\tau_j$ . Thus, for  $\forall \tau_j \in \Gamma_i$ , the effective coverage of user i is

 $\sum_{\tau_j \in \Gamma_i} \sum_{k:sa_{i,k} \cap a^j \neq \emptyset} (tsa_{i,k} \cdot |sa_{i,k}|).$  The effective average cost of user i is defined as  $\frac{b_i}{\sum_{\tau_j \in \Gamma_i} \sum_{k:sa_{i,k} \cap a^j \neq \emptyset} (tsa_{i,k} \cdot |sa_{i,k}|)}.$  In each iteration of the winner selection phase, we calculate  $tsa_{i,k}$  by calling function Allocation(·) for each user  $i \in U, k = 1, 2, ..., 2^{|\Gamma_i|} - 1$ , and select the user with minimum effective average cost over the unselected user set  $U \setminus S$  as the winner until the winners' sensing time can meet the requirement of minimum sensing time of each task in  $\Gamma$ .

We denote  $\mathbf{TSA}'$  as the sensing time allocation matrix in the payment determination phase. For each winner  $i \in S$ , we execute the winner selection phase over  $U \setminus \{i\}$ , and the winner set is denoted by S'. We compute the maximum price that user i can be selected instead of each user in S'. We will prove that this price is a critical payment for user i later.

Next, we present the algorithm of function Allocation( $\cdot$ ) to calculate the sensing time allocation matrix. Note that the

```
Algorithm 2 : MLI
Input: task set \Gamma, user set U, bid profile \mathbf{B}
```

```
Output: winner set S, sensing time allocation TSA, payment
          profile p
          //Phase 1: Winner Selection
   1: S \leftarrow \varnothing; \mathbf{TSA} \leftarrow 0; \mathbf{TSA}' \leftarrow 0;
   2: for all \tau_i \in \Gamma do
                t^{'j} \leftarrow t^j;
   4: end for
  5: while \sum_{\tau_i \in \Gamma} t^{'j} \neq 0 do
                \mathbf{TSA} \leftarrow Allocation\left(\Gamma, U \backslash S, \mathbf{B}, \{t^{'1}, t^{'2}, ..., t^{'m}\}\right);
                i \leftarrow \arg\min_{h \in U \setminus S} \frac{b_h}{\sum_{\tau_j \in \Gamma_h} \sum_{k: sa_{h,k} \cap a^j \neq \emptyset} (tsa_{h,k} \cdot |sa_{h,k}|)};
                S \leftarrow S \cup \{i\};
                \begin{array}{c} \text{for all } \tau_j \in \Gamma_i \text{ do} \\ t^{'j} \leftarrow t^{'j} - \sum_{k: sa_{i,k} \cap a^j \neq \emptyset} tsa_{i,k}; \end{array}
  9:
 10:
 11:
 12: end while
          //Phase 2: Payment Determination
 13: for all i \in U do
                p_i \leftarrow 0;
 15: end for
 16: for all i \in S do
                U' \leftarrow U \setminus \{i\}, S' \leftarrow \emptyset, t''^j \leftarrow t^j;
 17:
                while \sum_{\tau_j \in \Gamma} t^{"j} \neq 0 do
 18:
                      \mathbf{TSA}' \leftarrow Allocation\left(\Gamma, U' \backslash S', \mathbf{B}, \{t''^{1}, t''^{2}, ..., t''^{m}\}\right);
i_{h} \leftarrow \arg\min_{h \in U' \backslash S'} \frac{b_{h}}{\sum_{\tau_{j} \in \Gamma_{h}} \sum_{k: sa_{h,k} \cap a^{j} \neq \emptyset} \left(tsa'_{h,k} \cdot |sa_{h,k}|\right)};
 19:
20:
                    \begin{split} S^{'} &\leftarrow S^{'} \cup \{i_h\}; \\ p_i &\leftarrow \max\{p_i, \frac{\sum_{\tau_j \in \Gamma_i} \sum_{k:sa_{i,k} \cap a^j \neq \emptyset} tsa_{i,k}^{'}}{\sum_{\tau_j \in \Gamma_{i_h}} \sum_{k:sa_{i_h,k} \cap a^j \neq \emptyset} tsa_{i_h,k}^{'}} b_{i_h}\}; \\ \textbf{for all } \tau_j &\in \Gamma_{i_h} \textbf{do} \\ t^{''j} &\leftarrow t^{''j} - \sum_{k:sa_{i_h,k} \cap a^j \neq \emptyset} tsa_{i_h,k}^{'}; \end{split}
21:
22:
23:
24:
25:
                end while
26:
27: end for
```

platform want to maximize the value provided by each user, i.e., the effective coverage of user i for all tasks in  $\Gamma_i$ . As illustrated in Algorithm 3, for each user  $i \in U$ , we allocate the sensing time to the sensing areas iteratively. In each iteration, we select the sensing areas with most overlaps of AoIs. Here, we need a uniform tiebreaking rule when there are multiple sensing areas with same number of overlaps (e.g. index the sensing areas according to the geographic positions of the centroids). The allocated time is the minimum of the residual sensing time of user i and all residual sensing time of all tasks overlapping the selected sensing area. The iterations terminate when we have allocated all sensing time of user i or there is no unallocated sensing area.

**Theorem 5.** MLI is computationally efficient, individually rational, truthful, and  $H_k$  approximate, where

$$K = \max_{i \in U} \sum_{\tau_i \in \Gamma_i} \sum_{k: sa_{i,k} \cap a^j \neq \emptyset} (tsa_{i,k} \cdot |sa_{i,k}|).$$

Proof. For the time complexity, finding the user with min-

# Algorithm 3: Allocation

**Input:** task set  $\Gamma$ , user set U'', bid profile **B**, residual sensing time set  $\mathcal{R}$ 

Output: sensing time allocation TSA 1: for all  $i \in U^{''}$  do  $\begin{array}{l} t_{i}^{'} \leftarrow t_{i}; \ SA_{i}^{'} \leftarrow SA_{i}; \ \{\overline{t^{'1}}, \overline{t^{'2}}, ..., \overline{t^{'m}}\} \leftarrow \mathcal{R}; \\ \textbf{while} \ t_{i}^{'} > 0 \ \text{and} \ SA_{i}^{'} \neq \emptyset \ \textbf{do} \end{array}$  $k \leftarrow \operatorname{argmax}_{k:sa_{:..'} \in SA_i'} |sa_{i,k'}|;$ 4:  $j \leftarrow \operatorname{argmin}_{j':\underline{a}^{j'} \cap sa_{i,k} \neq \emptyset} \overline{t'j'};$   $tsa_{i,k} \leftarrow \min\{\overline{t'^{j}}, t'_{i}\};$   $t'_{i} \leftarrow t'_{i} - tsa_{i,k};$ 5: 6: 7:  $\begin{array}{l} c_{i} & \ddots & c_{i} \\ SA_{i}^{'} \leftarrow SA_{i}^{'} \backslash \{sa_{i,k}\}; \\ \text{for all } \tau_{j} \in \Gamma_{i}s.t. \ a^{j} \cap sa_{i,k} \neq \emptyset \ \textbf{do} \\ \hline t^{'j} \leftarrow t^{'j} - tsa_{i,k}; \end{array}$ 8: 9. 10: end for 11: end while 12: 13: **end for** 

## TABLE II WINNER SELECTION OF MLI

	User 1			User2	User3	User4	User5
$t_i$	4			1	2	4	4
$b_i$	5		2	3	7	9	
	$tsa_{1,1}$	$tsa_{1,2}$	$tsa_{1,3}$	$tsa_{2,1}$	$tsa_{3,1}$	$tsa_{4,1}$	$tsa_{5,1}$
Round 1	4	0	0	1	2	4	4
Winner							
Round 2				1	2	1	2
Winner							
Round 3				1		1	0
Winner							

imum effective average cost takes  $O(n \cdot 2^{\varepsilon})$ . The function Allocation(·) also takes  $O(2^{\varepsilon})$  time. Hence, the while-loop (Lines 5-12) takes  $O(n^2 \cdot 2^{\varepsilon})$ . The running time of the whole auction is dominated by the for-loop (Lines 16-27), which is bounded by  $O(n^3 \cdot 2^{\varepsilon})$ . Since  $\varepsilon$  is a small positive integer, the time complexity of MLI is acceptable. The proofs for individual rationality and truthfulness are similar to those of MLS. Since the LISO problem is equivalent to the MSMC problem, the greedy algorithm illustrated in Algorithm 2 is also  $H_k$  approximate, where  $K = \max_{i \in U} \sum_{\tau_j \in \Gamma_i} \sum_k tsa_{i,k}, \forall sa_{i,k} \in$  $SA_i, sa_{i,k} \cap a^j \neq \emptyset$  is the maximum size of the elements in multi-set collection T.

Remark: Note that we analyze the time complexity of MLI by considering both AoIs and active areas of users can be of any shape. In many crowdsensing systems, the number of sensing areas of a user is much smaller than  $2^{\varepsilon} - 1$  in practice. In the traffic monitoring crowdsensing system illustrated in Figure 1, the sensing areas of any user only exist on the crossroads, which is at most m-1. In this scenario, the time complexity of *MLI* is  $O(n^3 \cdot m)$  in fact.

## B. A Toy Example

We use an example in Figure 5 to show how *MLI* works.

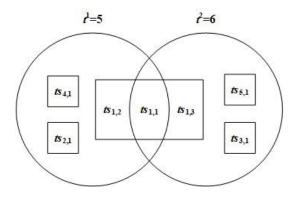


Fig. 5. Illustration for MLI, where there two spatio-temporal tasks represented by the circles with  $t^1 = 5$ ,  $t^2 = 6$ , respectively. There are five users represented by the rectangles with seven sensing areas. The maximum sensing time and bidding price are given in Table 2.

We directly give the sensing time of each round calculated by Algorithm 3 in Table 2, due to the space limitations.

Winner Selection:

Round 1: 
$$t^{'1} = 5$$
,  $t^{'2} = 6$ ,  $S = \emptyset$ 

$$\frac{b_1}{2tsa_{1,1}} = \frac{5}{8}, \ \frac{b_2}{tsa_{2,1}} = 1, \ \frac{b_3}{tsa_{3,1}} = \frac{3}{2}, \ \frac{b_4}{tsa_{4,1}} = \frac{7}{4}, \ \frac{b_5}{tsa_{5,1}} = \frac{9}{4}.$$

Round 2: 
$$t^{'1} = 5 - 4 = 1$$
,  $t^{'2} = 6 - 4 = 2$ ,  $S = \{1\}$   $\frac{b_2}{tsa_{2,1}} = 2$ ,  $\frac{b_3}{tsa_{3,1}} = \frac{3}{2}$ ,  $\frac{b_4}{tsa_{4,1}} = 7$ ,  $\frac{b_5}{tsa_{5,1}} = \frac{9}{2}$ .

Round 3: 
$$t^{'1}=1,\;t^{'2}=2-2=0,\;S=\{1,3\}$$
  $\frac{b_2}{tsa_{2,1}}=2,\;\frac{b_4}{tsa_{4,1}}=7.$  Thus  $S=\{1,3,2\}.$  For the payment determination, we directly give winners

when user i is excluded from the consideration, and only give the calculation for winner 1, due to the space limitations.

## Payment Determination:

For winner 1, winners are 3, 4, 2, 5 orderly.

• 
$$t''^1 = 5$$
,  $t''^2 = 6$ ,  $\frac{2 \times 4}{tsa_{3,1}} \times b_3 = \frac{2 \times 4}{2} \times 3 = 12$ 

For winner 1, winners are 3, 4, 2, 5 orderly.

• 
$$t''^1 = 5$$
,  $t''^2 = 6$ ,  $\frac{2\times 4}{tsa_{3,1}} \times b_3 = \frac{2\times 4}{2} \times 3 = 12$ 

•  $t''^1 = 5$ ,  $t''^2 = 6 - 2 = 4$ ,  $\frac{2\times 4}{tsa_{4,1}} \times b_4 = \frac{2\times 4}{4} \times 7 = 14$ 

•  $t''^1 = 5 - 4 = 1$ ,  $t''^2 = 4$ ,  $\frac{2\times 1 + 1\times 3}{tsa_{2,1}} \times b_2 = \frac{5}{2} \times 2 = 10$ 

•  $t''^1 = 1 - 1 = 0$ ,  $t''^2 = 4$ ,  $\frac{1\times 4}{tsa_{5,1}} \times b_5 = \frac{4}{4} \times 9 = 9$ 

• Thus  $p_1 = \max\{12, 14, 10, 9\} = 14$ 

• 
$$t''^1 = 5 - 4 = 1$$
,  $t''^2 = 4$ ,  $\frac{2 \times 1 + 1 \times 3}{t \cdot s \cdot a_{2,1}} \times b_2 = \frac{5}{2} \times 2 = 10$ 

• 
$$t''^1 = 1 - 1 = 0$$
,  $t''^2 = 4$ ,  $\frac{1 \times 4}{tsa_{5,1}} \times b_5 = \frac{4}{4} \times 9 = 9$ 

For winner 2, winners are 1, 3, 4 orderly.

• 
$$t''^1 = 5$$
,  $t''^2 = 6$ ,  $\frac{1}{2tsa_{1,1}} \times b_1 = \frac{5}{8}$ 

• 
$$t''^1 = 5$$
,  $t''^2 = 6$ ,  $\frac{1}{2tsa_{1,1}} \times b_1 = \frac{5}{8}$   
•  $t''^1 = 5 - 4 = 1$ ,  $t''^2 = 6 - 4 = 2$ ,  $\frac{1}{tsa_{3,1}} \times b_3 = \frac{3}{2}$   
•  $t''^1 = 1$ ,  $t''^2 = 2 - 2 = 0$ ,  $\frac{1}{tsa_{4,1}} \times b_4 = 7$ 

• 
$$t''^1 = 1$$
,  $t''^2 = 2 - 2 = 0$ ,  $\frac{1}{tsa_{4,1}} \times b_4 = 7$ 

• Thus  $p_2 = 7$ 

For winner 3, winners are 1, 2, 5 orderly.

• 
$$t''^1 = 5$$
,  $t''^2 = 6$ ,  $\frac{2}{2t \text{ sat.}} \times b_1 = \frac{5}{4}$ 

• 
$$t''^1 = 5$$
,  $t''^2 = 6$ ,  $\frac{2}{2tsa_{1,1}} \times b_1 = \frac{5}{4}$   
•  $t''^1 = 5 - 4 = 1$ ,  $t''^2 = 6 - 4 = 2$ ,  $\frac{2}{tsa_{2,1}} \times b_2 = 4$   
•  $t''^1 = 1 - 1 = 0$ ,  $t''^2 = 2$ ,  $\frac{2}{tsa_{5,1}} \times b_5 = 9$ 

• 
$$t''^1 = 1 - 1 = 0$$
,  $t''^2 = 2$ ,  $\frac{2}{tsa_{5,1}} \times b_5 = 9$ 

• Thus  $p_3 = 9$ 

# V. PERFORMANCE EVALUATION

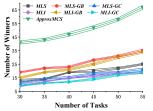
We have conducted thorough simulations to investigate the performance of MLS and MLI, and compare them with following five benchmark mechanisms: (1) MLS-GB: greedily select the user with minimum bidding price as the winner in the location sensitive model. (2) *MLS-GC*: greedily select the user with maximal effective coverage as the winner in the location sensitive model. (3) *MLI-GB*: greedily select the user with minimum bidding price as the winner in the location insensitive model. (4) *MLI-GC*: greedily select the user with maximal effective coverage as the winner in the location insensitive model. (5) *ApproxMCS*: truthful approximation algorithm [11] to find a sensing time schedule for maximizing the revenue of owner in mobile crowdsensing, where each user is able to perform at most one sensing task. All the simulations were run on a Centos 7 machine with Intel(R) Xeon(R) CPU E5-2630 2.6GHz and 128 GB memory. Each measurement is averaged over 100 instances.

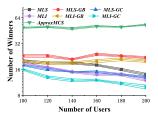
# A. Simulation Setup

We use the air pollution data [20] from the sites in Beijing and the T-Drive trajectory data [21], which contains trajectories of 10,357 taxis in Beijing, with geographic coordinates at different time for every trajectory. We randomly choose the sites and the taxis as the tasks and users, respectively. We set large AoIs to build the overlaps in metropolitan city of 16.41 thousand square kilometers. We consider that the AoI of each task is a circular region with radius uniformly distributed over [20, 110] kilometers. The task's sensing time is uniformly distributed over [5, 15] time units. The default number of tasks and users are 40 and 140, respectively. In MLS, we randomly choose one coordinate from taxi trajectory as user's location. In MLI, we regard the taxi trajectory between 14:30:29 and 15:00:29 as the active areas of the user. The maximum sensing time and cost of users are uniformly distributed over [5. 10] and [6, 10], respectively. We will vary the value of key parameters to explore the impacts of these parameters.

# B. Number of Winners

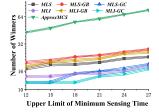
Figure 6 shows the number of winners of all mechanisms with different numbers of tasks and users, the minimum sensing time of tasks, and radius of AoIs. We can see that the winners increase with the numbers of tasks and minimum sensing time (vary the upper limit of uniform distribution from 12 to 27) of tasks since we need to select more users to complete the tasks. The winners of MLS, MLS-GC, MLI, and MLI-GC decrease with the number of users. This is because that the platform can select users with greater effective coverage when the number of users increases. However, the number of winners of MLS-GB and MLI-GB keep stable since they don't select the users based on the effective coverage. The winners of all incentive mechanisms but ApproxMCS decrease with increasing radius of AoI (vary the upper limit of uniform distribution from 100 to 150). Obviously, when the number of overlaps increases, the users will locate in the overlaps with a higher probability, improving the effective coverage of users. The winners of ApproxMCS keep stable with different numbers of users and radius of AoIs since each user only can participate in at most one task, and the number of tasks doesn't change. In all cases, ApproxMCS needs more users than *MLS* and *MLI* based incentive mechanisms since each user only can participate in at most one task in *ApproxMCS*. The winners of *MLS-GC* and *MLI-GC* are less than those of *MLS* and *MLI*, respectively. This is because *MLS-GC* and *MLI-GC* greedily select the user with maximal effective coverage. On the contrary, the winners of *MLS-GB* and *MLI-GB* are more than those of *MLS* and *MLI*, respectively, since *MLS-GC* and *MLI-GC* select the users only based on their bidding price.

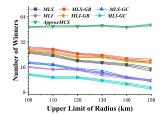




(a) Number of winners VS number of tasks

(b) Number of winners VS number of users





(c) Number of winners VS minimum sensing time of task

(d) Number of winners VS radius of AoI

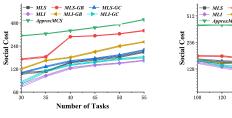
# C. Social Cost

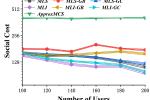
Fig. 6. Number of winners

Figure 7 shows the social cost of all incentive mechanisms. The trend of social cost is similar to that of number of winners shown in Figure 6 since the cost of users follows uniform distribution. Note that the *MLS* can output the solution with guaranteed approximation for our *LSSO* problem, and outperforms the *MLS-GB* and *MLS-GC* in all cases. The social cost of *MLS-GC* is very close to *MLS*. However, *MLS-GC* is untruthful. *MLI* outperforms the *MLI-GB* and *MLI-GC* in all cases too. Specifically, *MLS* outputs 22.3% and 5.3% less social cost than *MLS-GB* and *MLS-GC* on average, respectively. *MLI* outputs 33.6% and 7.8% less social cost than *MLI-GB* and *MLI-GC* on average, respectively.

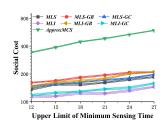
# D. Running Time

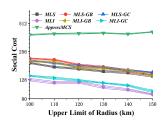
Figure 8 depicts the running time of all incentive mechanisms. It can be seen that the running time increases with the numbers of tasks, users and the minimum sensing time of tasks. The result is consistent with our time complexity analysis in Section 3 and Section 4. The running time of all incentive mechanisms decreases with the increase of radius of AoI becasue the number of winner decreases when the number of overlaps increases. Note that all incentive mechanisms iterate the winners in the payment determination phase, thus less time is needed. The running time of MLS-GB is much smaller than those of MLS and MLS-GC. This is because MLS-GB doesn't need to calculate the effective coverage, which





- (a) Social cost VS number of tasks
- (b) Social cost VS number of users

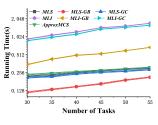


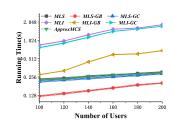


- (c) Social cost VS minimum sensing time
- (d) Social cost VS radius of AoI

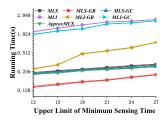
Fig. 7. Social cost

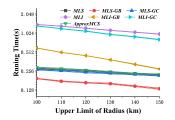
dominates the time complexity of *MLS* and *MLS-GC*. The running time of *MLS* is more than that of *MLS-GC* since there are fewer winners in *MLS-GC*. The similar analysis can be applied to *MLI*, *MLI-GB*, and *MLI-GC*.





- (a) Running time VS number of tasks
- (b) Running time VS number of users





- (c) Running time VS minimum sensing time of task
- (d) Running time VS radius of AoI

Fig. 8. Running time

# VI. RELATED WORK

Feng *et al.* proposed an incentive mechanism for mobile crowdsensing, where each task is with a sensing position, and each user has its sensing area [8]. The sensing areas can have overlaps, but the sensing data serves for single task exclusively. Moreover, the tasks don't have the time property.

Xu et al. proposed the incentive mechanisms for time window dependent tasks in mobile crowdsensing [12, 13],

where every task requires the sensing data in a specific time period. In [11], Han *et al.* proposed a truthful scheduling mechanism for powering mobile crowdsensing, where every task needs the sensing data of multiple time units. However, they assume that each user only can participate in one task. All above works focus on the time dependent tasks, and the location property of task is neglected.

In [9], the task specifies what sensor readings to report, and when and where to sense. If the sensing data satisfies the task requirement, the sensing data can be used to complete the task. In [10], smartphone users need to provide location-based services. The mobile crowdsensing app utilizes place detection to determine whether the user is at specific location. Wang *et al.* proposed a heterogeneous incentive mechanism [22], where the measurements of sensing task need to be executed at specific location during the specific time period. However, all above works consider that the tasks require the sensing data at the specific locations rather than the specific areas.

Overall, the issues of overlaps between the sensing areas and the corresponding time allocation haven't been studied yet. There is no off-the-shelf incentive mechanism designed in the literature for the crowdsensing system modeled in this paper.

## VII. CONCLUSION

In this paper, we have designed the incentive mechanisms for the mobile crowdsensing system with spatio-temporal tasks. We have presented two system models, location sensitive model and location insensitive model, to generalize the new scenario. We have designed two incentive mechanisms: *MLS* and *MLI*, to solve the problem of minimizing the social cost subject to the constraint that each task can be completed with the collective sensing time no less than the required minimum sensing time for the two models, respectively. Moreover, we have presented a sensing time allocation algorithm to allocate the sensing time of users to different sensing areas. Through both theoretical analysis and simulations, we have demonstrated that the proposed incentive mechanisms achieve computational efficiency, individual rationality, truthfulness, and guaranteed approximation.

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