Xu J, Fu JR, Yang DJ *et al.* FIMI: A constant frugal incentive mechanism for time window coverage in mobile crowdsensing. JOURNAL OF COMPUTER SCIENCE AND TECHNOLOGY 32(5): 919–935 Sept. 2017. DOI 10.1007/s11390-017-1773-4

FIMI: A Constant Frugal Incentive Mechanism for Time Window Coverage in Mobile Crowdsensing

Jia Xu[1,2], *Member, CCF, ACM, IEEE*, Jian-Ren Fu[1], De-Jun Yang[3], *Member, IEEE*, Li-Jie Xu[1,2], Lei Wang[1,2] and Tao Li[1,2], *Member, IEEE*

[1] *School of Computer Science, Nanjing University of Posts and Telecommunications, Nanjing 210023, China*

[2] *Jiangsu Key Laboratory of Big Data Security and Intelligent Processing, Nanjing University of Posts and Telecommunications, Nanjing 210023, China*

[3] *Department of Computer Science, Colorado School of Mines, Golden 80401, U.S.A.*

E-mail: xujia@njupt.edu.cn; 1039844431@qq.com; djyang@mines.edu; {ljxu, leiwang, towerlee}@njupt.edu.cn

Received February 17, 2017; revised July 27, 2017.

Abstract Mobile crowdsensing has become an efficient paradigm for performing large-scale sensing tasks. An incentive mechanism is important for a mobile crowdsensing system to stimulate participants and to achieve good service quality. In this paper, we explore truthful incentive mechanisms that focus on minimizing the total payment for a novel scenario, where the platform needs the complete sensing data in a requested time window (RTW). We model this scenario as a reverse auction and design FIMI, a constant frugal incentive mechanism for time window coverage. FIMI consists of two phases, the candidate selection phase and the winner selection phase. In the candidate selection phase, it selects two most competitive disjoint feasible user sets. Afterwards, in the winner selection phase, it finds all the interchangeable user sets through a graph-theoretic approach. For every pair of such user sets, FIMI chooses one of them by the weighted cost. Further, we extend FIMI to the scenario where the RTW needs to be covered more than once. Through both rigorous theoretical analysis and extensive simulations, we demonstrate that the proposed mechanisms achieve the properties of RTW feasibility (or RTW multi-coverage), computation efficiency, individual rationality, truthfulness, and constant frugality.

Keywords crowdsensing, incentive mechanism, constant frugality

1 Introduction

Smartphones have been widely available in recent years. According to the forecast of the International Data Corporation (IDC) in Jun. 2017, the worldwide smartphone market will reach a total of 1.51 billion units shipped in 2017①. Nowadays, smartphones are integrated with a variety of sensors such as camera, light sensor, GPS, accelerometer, digital compass, gyroscope, microphone, and proximity sensor. These sensors can collectively monitor a diverse range of human activities and surrounding environment.

Compared with the traditional sensor network, mobile crowdsensing[1-3], which utilizes hundreds of thousands of ordinary users[4], has a huge potential due to its prominent advantages, such as wide spatio-temporal coverage, low cost, good scalability, and pervasive application scenario. It will be an efficient approach to meeting the demand in large-scale sensing applications if we take advantage of pervasive smartphones to collect data.

Regular Paper

Special Section on Crowdsourced Data Management

The work was supported by the National Natural Science Foundation of China under Grant Nos. 61472193 and 61472192, the Young Scientists Fund of the National Natural Science Foundation of China under Grant No. 61502251, the Major Research Plan of the National Natural Science Foundation of China under Grant No. 91646116, the Natural Science Foundation of USA under Grant Nos. 1444059, 1420881 and 1717315, and the Natural Science Foundation of Jiangsu Province of China under Grant Nos. BK20141429, BK20151511 and BE2016776.

①Scarsella A, Stofega W. Worldwide smartphone forecast update, 2017-2021. http://www.idc.com/getdoc.jsp?containerId=US42776317, Aug. 2017.

Mobile crowdsensing can enable attractive sensing applications in various domains, such as Haze Watch② for pollution monitoring, Ear-Phone[5] for creating noise maps, RUSH[6] for real-time urban traffic speed estimation, FTrack[7] for floor localization, crowd-participated system[8] for bus arrival time prediction, and C^{2}IL[9] for Wi-Fi indoor localization.

The incentive mechanisms are crucial for mobile crowdsensing systems to compensate participants' resource consumption and potential privacy breach. The incentive mechanisms also help to achieve good service quality since sensing services are truly dependent on the quantity of users and the quality of sensed data. Numerous efforts have been made to develop incentive mechanisms for mobile crowdsensing. In offline mechanisms[10-12], the concurrent presence of numerous smartphone candidates is required. These offline schemes assume that all users are present for bidding before the task distribution, while online mechanisms[13-14] aim to deal with the case where users submit their profiles on the fly when they arrive.

However, these incentive mechanisms cannot deal with the time window coverage tasks which require completing sensing data in the whole time window publicized by the platform. There are some realistic examples of existing systems that fall into this scenario. One example is bus arrival time prediction system[8]. The users on the bus sense and submit the cell tower sequences to the backend server. Then the backend server predicts the bus' arrival time at various bus stops by matching the cell tower sequences to the bus route stored in the database. However, one contributor cannot always stay on one bus to collect information in a long time. Insufficient submitted information may result in inaccuracy in matching the bus route. The information assembling strategy is required to assemble pieces of incomplete information from multiple users to picture the intact bus route status. Another example is Ear-Phone[5], which is an end-to-end participatory urban noise mapping system. Noise levels are assessed on the mobile phones before being transmitted to the central server, where the noise map is reconstructed based on the partial noise measurements. The mobile phone computes a loudness characteristic known as the equivalent noise level over a specific time interval from the raw acoustic samples collected by the microphone. Then the central server computes the long-term equivalent noise level from the equivalent noise levels measured over short-time durations. A similar project for measuring air pollution in Australia is called Haze Watch②, where the continuous pollution readings are needed to cover the whole time line.

In aforementioned crowdsensing systems, the platform wants to collect sensing data over short-time durations sent by participants, and assembles pieces of incomplete information to reconstruct or represent the data over a long-time window. In essence, the platform needs to make sure that the submitted data over short-time window is sufficient for assembling in a long-time window. However, most existing incentive mechanisms consider the scenarios with the weak requirement of data integrity and cannot be applied to these crowdsensing systems. For example, the mechanisms in [10-13] deal with the location dependent tasks, and aim to optimize the payoff of the platform[10] or the values of the selected users' services under budget constraint[13-15] regardless of whether all tasks are performed.

In the system model of this paper, the platform wants to collect the continuous sensing data in the whole required time window (RTW), and each user responds with a user time window (UTW) in which the user can perform the tasks. The user time windows together are required to cover the request time window. As shown in Fig.1, the scenario of time window coverage mobile crowdsensing is very practical and pervasive especially when collecting time varying sensing data such as the continuous monitoring of traffic, noise, pollution and even the observation of garbage classification. Generally speaking, the time window coverage

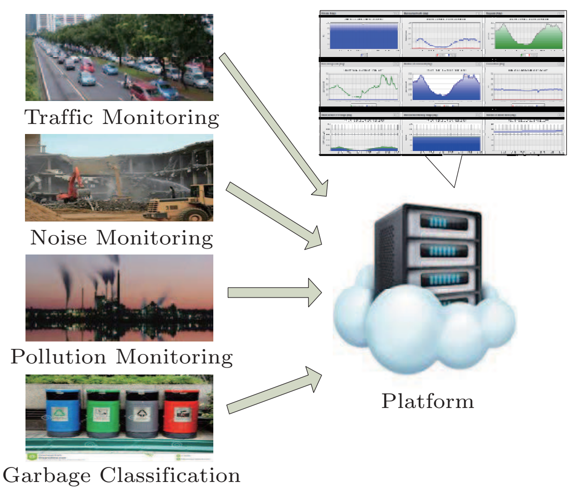


Fig.1. Time window coverage mobile crowdsensing.

②Carrapetta J, Youdale N, Chow A, Sivaraman V. Haze Watch project. http://www.hazewatch.unsw.edu.au, Aug. 2017.

tasks can be regarded as a long-term task, which lasts for the whole RTW and is unlikely to be accomplished by single human being, such as sampling the cell tower sequences along the whole bus route[8], measuring the long-term equivalent noise levels[9], and gathering the air pollution readings all the time.

We aim to design truthful incentive mechanisms to minimize the total payment for the time window coverage in mobile crowdsensing. The payment minimization problem is very challenging. First, the smartphone users would adopt strategic behaviors to maximize their own payoffs. For example, the strategic users can claim the bid price which is different from the real cost or report the time windows that are not real. Second, the mechanism of user selection should satisfy the property of RTW feasibility (i.e., the selected UTWs together can cover the RTW). This means that the mechanism should observe the sequential relationships between the UTWs and find an RTW feasible solution. Moreover, the payment minimization problem is challenging since the payment to the user might be different from its bid price in truthful mechanisms. In this paper, we design a constant frugal incentive mechanism for time window coverage, termed FIMI, based on frugality theory[16-17].

Our key contributions are summarized as follows.

• We propose the system model for time window coverage mobile crowdsensing and formulate the problem of payment minimizing user selection (PMUS).

• We design FIMI based on the frugality theory to solve the PMUS problem. To the best of our knowledge, this is the first work to design incentive mechanisms of mobile crowdsensing systems based on the frugal theory. We show that FIMI satisfies five desirable properties: RTW feasibility, computational efficiency, individual rationality, truthfulness, and constant frugality.

• We further extend FIMI to support RTW multi-coverage requirement. We show that the extended FIMI also satisfies the five desirable properties.

• We extensively evaluate the performance of FIMI based on both real trace and randomly generated users. Our evaluation results show that FIMI can achieve less payment than VCG (Vickrey-Clarke-Groves) auction[18].

The rest of the paper is organized as follows. We review related work in Section 2. Section 3 formulates the system model and problem. Section 4 reviews some related solution concepts and presents the detailed design of FIMI. Section 5 extends FIMI to support RTW multi-coverage requirement. Performance evaluation is presented in Section 6, and Section 7 concludes this paper.

2 Related Work

Many task allocation algorithms have been proposed. Zheng *et al.* explored how accuracy and F-score, two widely-used evaluation metrics for crowdsourcing applications, can facilitate task assignment[19]. An adaptive crowdsourcing framework, called iCrowd, was proposed in [20]. iCrowd on-the-fly estimates the accuracy of a worker by evaluating his/her performance on the completed tasks, and predicts which tasks the worker is well acquainted with. Hu *et al.* proposed a crowdsourcing framework consisting of an inference model and an online task assigner[21]. However, the above work cannot be applied to the time window coverage tasks.

Kazemi *et al.* focused on the spatial task allocation problem[22-23]. In the scenario of [22], the server assigns to every worker his/her nearby tasks with the objective of maximizing the overall number of assigned tasks. Tong *et al.* addressed the global online micro-task allocation in spatial crowdsourcing (GOMA) problem based on bipartite graph matching[23]. Tong *et al.* presented a comprehensive experimental comparison of the representative algorithms of the online minimum bipartite matching (OMBM) problem[24]. Overall, the above studies, which aim to solve the spatial task or service provider allocation problem, cannot solve the time interval coverage problem. Moreover, this paper utilizes the reward based incentive, and tries to propose a truthful incentive mechanism for the selfish users. Therefore the techniques proposed in [22-24] cannot be used to solve the problem of this paper.

Many incentive mechanisms for mobile crowdsensing have been proposed thus far. Singer proposed a truthful budget feasible mechanism[15] based on the proportional share allocation rule. However, the designed mechanism is not established on any crowdsensing system model and only valid for submodular functions. Pricing mechanisms are also developed in [11] for the budget feasible maximizing task problem and the budget feasible minimizing payment problem based on the method proposed in [15]. Yang *et al.* proposed two different models for smartphone crowdsourcing[10]: the platform-centric model where the platform provides a reward shared by participating users, and the user-centric model where users have more control over the payment they will receive. In [25], they further extended the user-centric model to three cases: single requester with single bid, single requester with multiple bids, and multiple requesters with multiple bids. Xu *et al.* designed the incentive mechanisms, which consider the issue of stimulating the biased requesters in the competing crowdsourcing market[26]. Koutsopoulos designed an optimal reverse auction[27], considering the data quality as user participation level. However, the quality indicator, which essentially measures the relevance or usefulness of information, is empirical and relies on users' historical information. It is not reasonable to assume that the historical information can be obtained in advance. In [12], Feng *et al.* formulated the location-aware collaborative sensing problem as the winning bids determination problem, and presented a truthful auction using the proportional share allocation rule proposed in [15]. However the mechanism is only effective to perform location-aware tasks. In [13], Zhao *et al.* investigated the online crowdsourcing scenario where the users submit their profiles to the crowdsourcer when they arrive. The objective is selecting a subset of users for maximizing the value of the crowdsourcer under the budget constraint. They designed two online mechanisms, online mechanism under zero arrival-departure interval model (OMZ) and online mechanism under general arrival-departure interval model (OMG) for different user models. Xu *et al.* also tackled the mobile crowdsensing with strong requirement of data integrity[28-29]. However, [28] focuses on optimizing the social cost, and [29] focuses on maximizing the value of platform under budget constraint.

A theory of frugality has been developed with the goal of providing mechanisms that admit minimal payment. In [16], Archer and Tardos proposed the frugality problem in path auction, and analyzed the overpayment of VCG mechanism. As observed by Talwar[30], the class of instances for which VCG never overpays is a natural generalization of matroids. Moreover, some sufficient conditions to upper bound and lower bound of the overpayment in non-matroids cases were given in [30]. The benchmark of payment for any truthful mechanism and the concept of frugality ratio were proposed in [17]. Kempe *et al.* studied truthful mechanisms for hiring a team of agents in three classes of set systems: vertex cover auctions, k-flow auctions and cut auctions, demonstrating the truthful mechanisms for all three set systems are constant-competitive[31]. Elkind *et al.* proposed a modified definition of frugality and a truthful polynomial-time auction for the vertex cover problem[32]. Chen *et al.* proposed a uniform scheme for designing frugal truthful mechanisms for general set systems, and applied the scheme to vertex cover systems and k-path systems[33].

Overall, there is no efficient method to solve the payment minimizing problem in mobile crowdsensing when the distribution of cost is unknown. Designing incentive mechanisms, which achieve truthfulness and constant frugality simultaneously, is an open problem that none of the proposals has solved.

3 System Model and Problem Formulation

As illustrated in Fig.2, we consider a mobile crowdsensing system consisting of a platform and a set of smartphone users $U = \{1, 2, \ldots, n\}$. The platform publicizes a required time window (RTW) $\mathcal{W} = [T_{\rm S}, T_{\rm E}]$, where $T_{\rm S}$ and $T_{\rm E}$ are the start time and the end time, respectively. The platform requests the sensing data in the period from $T_{\rm S}$ to $T_{\rm E}$. We denote the length of RTW, i.e., the number of time unit, as $|\mathcal{W}|$. The time unit, which is closely related to the application scenario, is determined by the sampling frequency of sensing data in practice. Each user i responds with a bid $B_i = ([s_i, e_i], b_i)$, where $[s_i, e_i]$ is the user time window (UTW) within which user i can perform. Each UTW $[s_i, e_i]$ is associated with cost c_i. The start time s_i and the end time e_i ($s_i \leqslant e_i$) can be any point-in-time. However, any $s_i < T_{\rm S}$ or $e_i > T_{\rm E}$ cannot bring extra benefit for user i in our mechanism. b_i is the claimed cost which is the bid price that user i wants to charge for performing $[s_i, e_i]$. For any subset of users $S \subseteq U$, let $c(S) = \sum_{i \in S} b_i$. We consider the real cost c_i is private and unknown to other users and the platform.

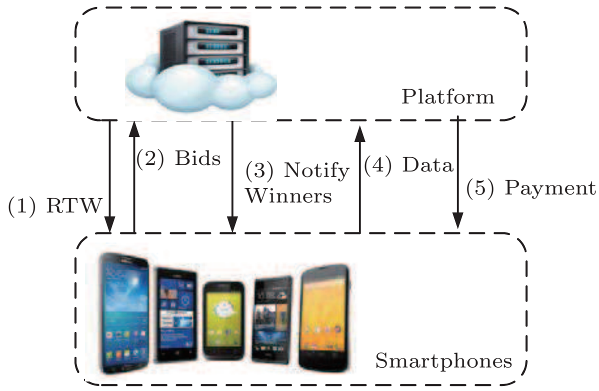


Fig.2. Illustration of a mobile crowdsensing system. The number represents the sequence of the interactions between the platform and users.

The platform selects a subset of users $S \subseteq U$ as winners and notifies them. The winners perform the sensing tasks in their UTWs and send data back to the platform. Each user i is then paid p_i , which is computed by the platform.

We define the utility of user i as the difference between the received payment and its real cost: $u_i = p_i - c_i$. Specially, the utility of the losers would be zero, because they are unpaid in our designed mechanism and there is no cost for sensing.

We consider that the users are selfish individuals and may adopt strategic behavior by claiming cost that might be different from the real cost to maximize their own utility.

The users can also take a strategic behavior by reporting time windows that are not real. This strategic behavior can be prevented if the platform can verify whether all sensing data in announced time windows is submitted and whether the sensing data is generated at the announced time. For this purpose, we assume the sensing data is processed by trusted time stamping such as Public Key Infrastructure Time-Stamp Protocol (TSP)③.

The utility of the platform is

$$u_0 = v(\mathcal{W}) - \sum_{i \in S} p_i, \tag{1}$$

where $v(\mathcal{W})$ is the value to the platform when it obtains all data in the whole $\mathcal{W}$.

For any $\mathcal{W}$ and strategy bids $\boldsymbol{B} = (B_1, ..., B_n)$, we consider an incentive mechanism $\mathcal{M}(\mathcal{W}, \boldsymbol{B})$ returning a subset of users $S \subseteq U$ and a payment vector $\boldsymbol{p} = (p_1, ..., p_n)$ to all the users. The objective function is minimizing the total payment which is the sum of the payment of selected users for completing the sensing in the whole $\mathcal{W}$. The PMUS problem can then be formulated as:

$$\min \sum_{i \in S} p_i$$
$$\text{s.t. } \mathcal{W} \subseteq \cup_{i \in S}[s_i, e_i], \tag{2}$$

where operation $\cup$ returns the set of covered time units by each $[s_i, e_i]$.

The constraint in (2) means that the UTWs should cover the RTW, i.e., the mechanism should assure that the winners can perform all tasks from $T_{\rm S}$ to $T_{\rm E}$. This means that the value to the platform $v(\mathcal{W})$ is constant. Thus, minimizing the total payment is equivalent to maximizing the utility of the platform defined in (1). We assume that there are enough users and exclude the situation where only one bid hits the arbitrary time unit in $[T_{\rm S}, T_{\rm E}]$ in order to prevent the monopoly.

The participants can decide the UTWs based on their future schedules or daily mobility routines with little effect on their daily life. Much research has demonstrated that people show striking persistence in their mobility profiles. In [34], the authors stated that the similarity of the mobility profile of a given user to its future profile is above 0.75 for eight days and remains above 0.6 for five weeks, demonstrating the mobility profile is indeed an intrinsic property, even if only a short history of mobility profile is used.

Our objective is to design an incentive mechanism $\mathcal{M}(\mathcal{W}, \boldsymbol{B})$ satisfying the following five desirable properties.

1) *RTW Feasibility.* A mechanism $\mathcal{M}(\mathcal{W}, \boldsymbol{B})$ is RTW feasible if the UTWs of winners together can cover the whole RTW, i.e., the solution is feasible if it satisfies the constraint in (2).

2) *Computational Efficiency.* A mechanism $\mathcal{M}(\mathcal{W}, \boldsymbol{B})$ is computationally efficient if both the winner set S and the payment vector $\boldsymbol{p}$ can be computed in polynomial time.

3) *Individual Rationality.* Each user will have a non-negative utility, i.e., $p_i \geqslant c_i, \forall i \in U$.

4) *Truthfulness.* A mechanism is truthful if no user can improve its utility by submitting a bidding price different from its real cost, no matter what others submit. In other words, reporting the real cost is a dominant strategy[18] for all users.

5) *Constant Frugality.* The objective function is minimizing the total payment of the platform. We hope that the mechanism can use as small payment as possible for performing all tasks and still satisfy the aforementioned properties. We measure the amount "overpayment" of the truthful mechanisms by the frugality ratio defined in Section 4. We say that a mechanism is α-frugal if it has a frugality ratio within a factor of α of the optimal frugality ratio. Here, our goal is to design a truthful mechanism with a constant factor of optimality, namely, the mechanism can satisfy the constant frugality or sub-optimal frugality.

③Adams C, Pinkas D. Internet x.509 public key infrastructure time-stamp protocol (TSP), August 2001. https://tools.ietf.org/html/rfc3161, Aug. 2017.

4 FIMI

4.1 Related Solution Concept

Before presenting the design of our proposed FIMI incentive mechanism in details, let us review some important and useful solution concepts of truthfulness and frugality.

We first introduce Myerson's theorem[35].

Theorem 1[15]. *An auction mechanism is truthful if and only if*:

1) *the selection rule is monotone*: *if user i wins the auction by bidding b_i, it also wins by bidding $b'_i \leqslant b_i$*;

2) *each winner is paid the critical value*: *user i would not win the auction if it bids higher than this value.*

Definition 1 (Benchmark $v(\boldsymbol{c})$[17]). *Given a set system $(\mathcal{E}, \mathcal{F})$, and a feasible set $S \in \mathcal{F}$ of minimum total cost w.r.t. $\boldsymbol{c}$, let $v(\boldsymbol{c})$ be the value of an optimal solution to the following optimization problem*:

$$\min \sum_{i \in S} f_i$$

$$\begin{aligned} \text{s.t.}\quad & 1) f_i \geqslant c_i, \forall i \in \mathcal{E}, \\ & 2) \sum_{i \in S \setminus T} f_i \leqslant \sum_{j \in T \setminus S} c_j, \forall T \in \mathcal{F}, \\ & 3) \textit{for every } i \in S, \textit{ there is } \ T \in \mathcal{F} \\ & \quad \text{s.t.} \quad i \notin T \textit{ and } \sum_{j \in S \setminus T} f_j = \sum_{j \in T \setminus S} c_j, \end{aligned}$$

where $\mathcal{E}$ is the set of elements and $F \subseteq 2^{\mathcal{E}}$ is the collection of feasible sets. $f_i, i \in S$, is the cheapest payment to i when satisfying the above three constraints.

The first constraint states that the users are individual rational, and the second constraint ensures that set S indeed has the lowest total bid among feasible sets. With the third constraint, no winner $i \in S$ can improve his/her utility by increasing the bid, as he/she would not be a winner anymore.

Intuitively, in the optimal solution of the above system, S is the set of winners in the first-price auction[18]. $v(\boldsymbol{c})$ gives the value of the cheapest "Nash equilibrium" of the first-price auction assuming that the most "efficient" feasible set S wins.

Remark. It should be noted that first-price auctions do not in general have Nash equilibrium due to tie-breaking issues[36]. However, $v(\boldsymbol{c})$ is a very intuitive value, and indeed provides a bound on the total payment of any truthful mechanism in our system model.

Definition 2 (Frugality Ratio)[33]. *Let $\mathcal{M}$ be a truthful mechanism for the set system $(\mathcal{E}, \mathcal{F})$ and let $p_{\mathcal{M}}(\boldsymbol{c})$ denote the total payment of $\mathcal{M}$ when the true costs are given by vector $\boldsymbol{c}$. Then the frugality ratio of $\mathcal{M}$ on $\boldsymbol{c}$ is defined as $\emptyset_{\mathcal{M}}(\boldsymbol{c}) = \frac{p_{\mathcal{M}}(\boldsymbol{c})}{v(\boldsymbol{c})}$. Further, the frugality ratio of $\mathcal{M}$ is defined as $\emptyset_{\mathcal{M}} = sup_c \emptyset_{\mathcal{M}}(\boldsymbol{c})$.*

4.2 Design Rationale

It is not difficult to see that there is the optimal solution for minimizing social cost (the total cost of selected users) problem since we can structure an interval graph for all $[s_i, e_i]$, $i \in U$, with weighted vertexes, and find the shortest path from any vertex with $s_i \leqslant T_{\rm S}$ to any vertex with $e_i \geqslant T_{\rm E}$. This can be done in polynomial time. It is well-known that the VCG mechanism is the only truthful mechanism when selecting the optimal solution as the winners[16]. We give the frugality ratio of VCG auction in our PMUS problem.

Theorem 2. *The VCG mechanism has a frugality ratio of $(|S|)$ in PMUS problem.*

Proof. We consider the special case where there are two disjoint feasible subsets of users, S' and S'', with total cost $TC' = \sum_{i \in S'} b_i$ and $TC'' = \sum_{j \in S''} b_j$, and the number of users $|S'|$ and $|S''|$ respectively. If the winner set is S', i.e., $TC' \leqslant TC''$, then for each user $i \in S'$, the payment is $TC'' - TC' + b_i$ since this is the highest it can bid before S'' becomes the winner set. Thus the total payment to S' is $\sum_{i=1}^{|S'|}(TC'' - TC' + b_i) = |S'| \times (TC'' - TC') + TC'$. It is not difficult to find the cheapest Nash equilibrium and obtain $v(\boldsymbol{c}) = TC''$ in this case based on Definition 1. Therefore the frugality ratio is $\frac{|S'| \times (TC'' - TC') + TC'}{TC''} = \Omega(|S'|)$. On the contrary, the frugality ratio is $\Omega(|S''|)$ if S'' wins. □

In this paper, we resort to the frugal path auction[17], which incorporates techniques originated from the frugality theory[16], to solve our PMUS problem. Generally, frugal path auction consists of three steps: 1) finding two disjoint feasible paths; 2) finding interchangeable subpaths of the two paths, and 3) selecting the subpaths as the winners by the weighted cost. However, the time window is different from the path since the relationships between the UTWs can be very complex and there are no distinct sequential relationships between them. We need find two disjoint user sets from the UTWs and the interchangeable user set pairs from the two disjoint user sets. In addition, the payment rule should be designed carefully to achieve the individual rationality and truthfulness.

Inspired by [17], we extend the frugal path auction by performing the auction in two phases: candidate selection and winner selection. In the candidate selection phase, FIMI first maps the UTWs to the interval graph in order to expose the overlaps of UTWs. Then FIMI transforms the interval graph to the flow graph and finds the solutions using the minimum cost flow algorithm[37]. By this way, FIMI can obtain two disjoint user sets, while achieving the property of RTW feasibility. In the winner selection phase, to expose the structure of the competition, FIMI should find the interchangeable user set pairs between the two disjoint user sets obtained in the candidate selection phase. This can be achieved from the intersection relationships between the disjoint user sets on the flow graph. To achieve the constant frugality, FIMI considers the size (weight) of interchangeable user set pairs and selects the winners from such user set pairs by the weighted cost. Finally, FIMI sets the minimum value of payment calculated in the candidate selection phase and the winner selection phase. We will prove that the minimum value is the critical value in the designed mechanism, and the selection rules in both phases are monotone, guaranteeing the property of truthfulness. The processing workflow of FIMI is shown in Fig.3.

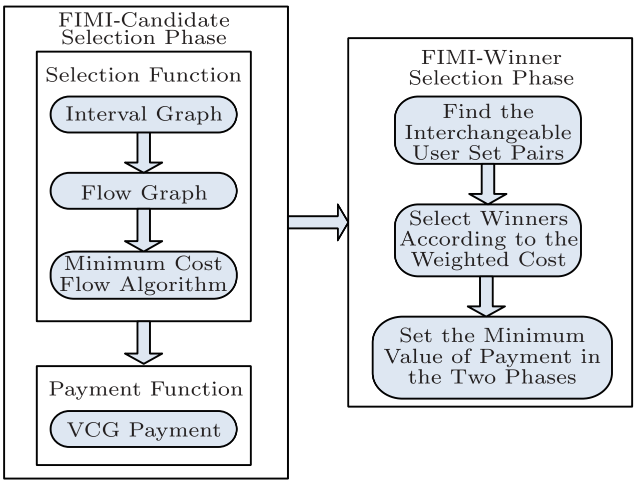


Fig.3. Processing workflow of FIMI.

4.3 Phase 1: Candidate Selection

FIMI first selects two disjoint feasible user sets S'_C and S''_C minimizing $\sum_{i\in S'_C} b_i + \sum_{j\in S''_C} b_j$, and calculates the critical payment for each user in S'_C and S''_C. Let $S_C = S'_C \cup S''_C$. As illustrated in Algorithm 1, we use the selection function to pick up the users, and the payment is calculated by the payment function.

The pseudo-code of the selection function is shown in Algorithm 2, which uses a graph-theoretic approach to obtain sets S'_C and S''_C. We introduce the functions used in Algorithm 2 here.

Algorithm 1. FIMI-Candidate Selection

Input: RTW $\mathcal{W}$, strategy bids $\boldsymbol{B}$, set of users U

$S_C \leftarrow \emptyset$;
$S_C \leftarrow \mathit{Selection}(\mathcal{W}, \boldsymbol{B}, U)$;
$\boldsymbol{p} \leftarrow \mathit{Payment}(\mathcal{W}, \boldsymbol{B}, U, S_C)$;
return $(S_C, \boldsymbol{p})$;

Algorithm 2. Selection

Input: RTW $\mathcal{W}$, strategy bids $\boldsymbol{B}$, set of users U

$S_C \leftarrow \emptyset$;
$G_I(V_I, E_I, \boldsymbol{w}) \leftarrow \mathit{IntervalGraph}(U, \boldsymbol{B})$;
$G_F(V_F, E_F, \boldsymbol{w}, \boldsymbol{a}, s, t) \leftarrow \mathit{FlowGraph}(G_I(V_I, E_I, \boldsymbol{w}))$;
$S_C \leftarrow \mathit{MinCostFlow}(G_F(V_F, E_F, \boldsymbol{w}, \boldsymbol{a}, s, t), 2)$;
return (S_C);

IntervalGraph$(U, \boldsymbol{B})$. For each user $i \in U$, we create a vertex $v_i \in V_I$ with weight $w_i = b_i$, and create an edge $(v_i, v_j) \in E_I$ if there is an overlap between $[s_i, e_i]$ and $[s_j, e_j]$.

Based on the interval graph, FIMI then constructs a flow graph $G_F(V_F, E_F, \boldsymbol{w}, \boldsymbol{a}, s, t)$, where $\boldsymbol{a}$ is the capacity for each edge in E_F, and s and t are the source and the sink respectively. This transformation process can be as follows.

FlowGraph$(G_I(V_I, E_I, \boldsymbol{w}))$. 1) For each $v_i \in V_I$, create two vertexes v'_i and v''_i into V_F. We create a directed edge (v'_i, v''_i) with $w(v'_i, v''_i) = b_i$, $a(v'_i, v''_i) = 1$ into E_F. 2) For each $(v_i, v_j) \in E_I$, create directed edge (v''_i, v'_j) with $w(v''_i, v'_j) = 0, a(v''_i, v'_j) = 1$ and directed edge (v''_j, v'_i) with $w(v''_j, v'_i) = 0, a(v''_j, v'_i) = 1$ into E_F. 3) Create vertexes s and t into V_F. We create a directed edge (s, v'_i) with $w(s, v'_i) = 0, a(s, v'_i) = 1$ into E_F if $T_{\mathrm{S}} \in [s_i, e_i]$. We create a directed edge (v''_i, t) with $w(v''_i, t) = 0, a(v''_i, t) = 1$ into E_F if $T_{\mathrm{E}} \in [s_i, e_i]$.

MinCostFlow$(G_F(V_F, E_F, \boldsymbol{w}, \boldsymbol{a}, s, t), 2)$. Find the minimum cost flow with value of 2 in G_F[37].

In this way, the output of function *MinCostFlow* can be decomposed into two edge-disjoint paths, A and A'.

To compute the payment, we apply the VCG payment rule, and each selected user will be paid an amount equal to the benefit it introduces to the system, i.e., the difference between other users' minimum total cost with and without it. The payment function is described in Algorithm 3.

Algorithm 3. Payment

Input: RTW $\mathcal{W}$, strategy bids $\boldsymbol{B}$, set of users U, set of selected users S_C

Let $p_i \leftarrow 0$ for all $i \in U$;
forall $i \in S_C$ **do**
  $S_C^{-i} \leftarrow \mathit{Selection}(\mathcal{W}, \boldsymbol{B}, U \backslash i)$;
  $p_i \leftarrow c(S_C^{-i}) - (c(S_C) - b_i)$;
return $(\boldsymbol{p})$;

4.4 Phase 2: Winner Selection

In this phase, we first find all the interchangeable user sets, who can replace one another for completing the tasks in the same time period. For every pair of such user sets, we choose one of them based on the weighted cost. The pseudo-code of winner selection is shown in Algorithm 4.

Algorithm 4. FIMI-Winner Selection

Input: flow graph G_F, set of selected users S_C, strategy bids $\boldsymbol{B}$, payment $\boldsymbol{p}$

$S_C \leftarrow \emptyset$;
Find the intersection points of A and A';
Let $s = d_1, d_2, ..., d_{k+1} = t$ be the intersection points of A and A' in the order they appear in A and A';
Let S_i (resp. S_i') be the user subset of S_C' (resp. S_C'') from d_i to d_{i+1};
for $i \leftarrow 1$ to k **do**
  if $\sqrt{|S_i|} \times c(S_i) \leqslant \sqrt{|S_i'|} \times c(S_i')$ **then**
    forall $j \in S_i$ **do**
      $S \leftarrow S \cup \{j\}$;
      $p_j \leftarrow \min\{p_j, \sqrt{|S_i'|} \times \frac{c(S_i')}{\sqrt{|S_i|}} - c(S_i) + b_j\}$;
  else
    forall $j \in S_i'$ **do**
      $S \leftarrow S \cup \{j\}$;
      $p_j \leftarrow \min\{p_j, \sqrt{|S_i|} \times \frac{c(S_i)}{\sqrt{|S_i'|}} - c(S_i') + b_j\}$;
Let $p_i \leftarrow 0$ for all $i \in U \backslash S$;
return $(S, \boldsymbol{p})$;

FIMI uses the following facts to find the interchangeable user sets.

Fact 1. *If there is an edge (v_i'', u_{j+1}') in G_F, where $v_i'' \in A, u_{j+1}' \in A'$, then the users before $i+1$ in A can replace the users before $j+1$ in A' for completing the tasks in $[T_{\rm S}, s_{j+1}]$.*

Fact 1 relies on the observation that the two UTWs have the overlap iff there is at least one edge between them in G_F.

Fact 2. *If there are two edges (v_i'', u_{j+1}') and (u_j'', v_{i+1}') of G_F, where v_i'' and v_{i+1}' are adjacent vertexes in A, and u_j'' and u_{j+1}' are adjacent vertexes in A', then the set of users before $i+1$ in A and the set of users before $j+1$ in A' are the interchangeable user sets.*

Fact 2 means that we can obtain the interchangeable user sets, which can be replaced by one another, by finding such two edges in G_F. Since such two user sets are interchangeable, we can combine $v_i'', u_{j+1}', u_j'', v_{i+1}'$ into one vertex, which is an intersection point of path A and path A'. Fig.4 illustrates how to find an intersection point in G_F.

FIMI finds all the intersection points. Let $s = d_1, d_2, ..., d_{k+1} = t$ be the intersection point sequence ordered by their occurrence in A and A'. Let S_i (resp. S_i') be the user subset of S_C' (resp. S_C'') from d_i to $d_{i+1}, i = 1, 2, ..., k$. Since S_i and S_i' are interchangeable user subsets, FIMI chooses one of them based on the weighted cost. The weight for user set S_i (resp. S_i') is $\sqrt{|S_i|}$ (resp. $\sqrt{|S_i'|}$), where $|S_i|$ is the number of users in S_i. We will show that this setting of weight can achieve constant frugality. The selection rule is: if $\sqrt{|S_i|} \times c(S_i) \leqslant \sqrt{|S_i'|} \times c(S_i')$, then S_i wins; otherwise, S_i' wins. As a result, the winners are the users in all winning user sets. In the end, we set the value of $p_j, j \in S$, to the minimum value of payment calculated in the candidate selection phase and the winner selection phase respectively.

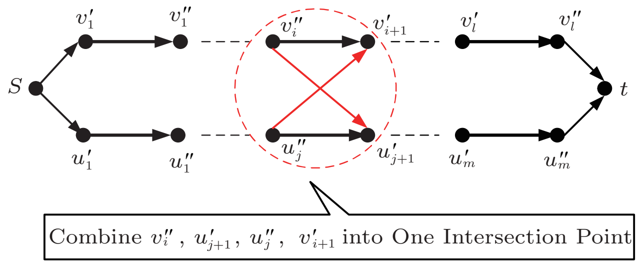


Fig.4. Illustration for finding an intersection point.

4.5 Walk-Through Example

We use the example in Fig.5 to illustrate how FIMI works, and compare the total payment with that of VCG auction.

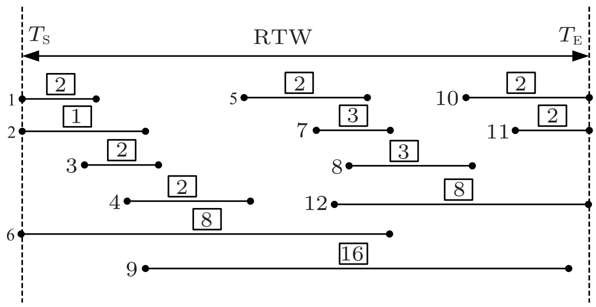


Fig.5. Illustration for FIMI. There are nine UTWs from different bidders. The number before a UTW is the user ID, and the square represents its bid.

- *VCG Auction.*

Selection. The winners are {2, 4, 5, 8, 3}, and the total cost is 10.

Payment. $p_2 = c(\{6, 8, 3\}) - 10 + b_2 = 4$, $p_3 = c(\{2, 4, 5, 7\}) - 10 + b_3 = 5$, $p_4 = c(\{6, 8, 3\}) - 10 + b_4 = 5$, $p_5 = c(\{6, 8, 3\}) - 10 + b_5 = 5$, $p_8 = c(\{2, 4, 5, 7\}) - 10 + b_8 = 6$. The total payment is 25.

• *FIMI*.

1) *Phase* 1: *Candidate Selection*.

Selection. Select two disjoint feasible subsets with minimum total cost. The winners are $\{2, 4, 5, 8, 3\}$ and $\{6, 7\}$ (or $\{2, 4, 5, 7\}$ and $\{6, 8, 3\}$), and total cost is 26. Note that the choice does not impact the solution and payment. The result is shown in Fig.6.

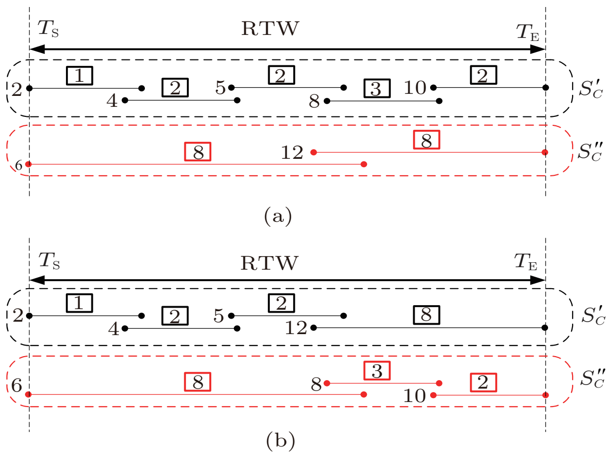


Fig.6. Two disjoint feasible user sets {2, 4, 5, 8, 3} and {6, 7} with minimum total cost. There is an alternative {2, 4, 5, 7} and {6, 8, 3} with the same total cost.

Payment. $p_2 = c(\{1,4,5,8,3,6,7\}) - 26 + b_2 = 5$, $p_3 = c(\{2,9,6,7\}) - 26 + b_3 = 9$, $p_4 = c(\{2,9,6,8,3\}) - 26 + b_4 = 6$, $p_5 = c(\{6,8,3,2,9\}) - 26 + b_5 = 6$, $p_6 = c(\{2,4,5,8,3,1,9\}) - 26 + b_6 = 13$, $p_7 = c(\{2,9,6,8,3\}) - 26 + b_7 = 12$, $p_8 = c(\{2,9,6,7\}) - 26 + b_8 = 10$.

2) *Phase* 2: *Winner Selection*.

The interchangeable user sets are shown in Fig.7.

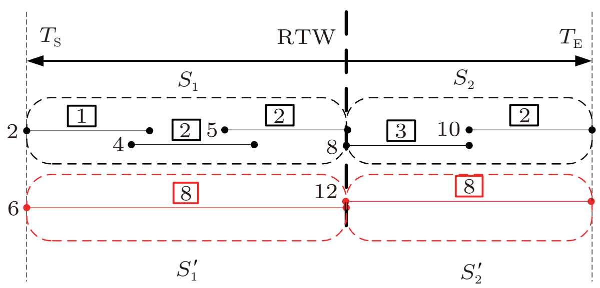


Fig.7. Selecting winners from interchangeable user sets S_i and S'_i where $S_1 = \{2, 4, 5\}$, $S'_1 = \{6\}, S_2 = \{8, 3\}, S'_2 = \{7\}$.

a) $S_1 = \{2, 4, 5\}$ vs $S'_1 = \{6\}$

$\sqrt{|S_1|} \times c(S_1) = \sqrt{3} \times 5 \geqslant \sqrt{|S'_1|} \times c(S'_1) = 1 \times 8$, S'_1 wins. $p_6 = \min\{p_6, \sqrt{|S_1|} \times \frac{c(S_1)}{\sqrt{|S'_1|}} - c(S'_1) + b_6\} = 5\sqrt{3}$.

b) $S_2 = \{8, 3\}$ vs $S'_2 = \{7\}$

$\sqrt{|S_2|} \times c(S_2) = \sqrt{2} \times 5 \leqslant \sqrt{|S'_2|} \times c(S'_2) = 1 \times 8$, S_2 wins. $p_8 = \min\{p_8, \sqrt{|S'_2|} \times \frac{c(S'_2)}{\sqrt{|S_2|}} - c(S_2) + b_8\} = \frac{8}{\sqrt{2}} - 2$; $p_3 = \min\{p_3, \sqrt{|S'_2|} \times \frac{c(S'_2)}{\sqrt{|S_2|}} - c(S_2) + b_3\} = \frac{8}{\sqrt{2}} - 3$.

The winners are $S'_1 \cup S_2 = \{6, 8, 3\}$. The total cost is $8 + 3 + 2 = 13$. The total payment is $p_6 + p_8 + p_3 = 8\sqrt{2} + 5\sqrt{3} - 5 \approx 14.97$, which is less than that of VCG auction.

4.6 Mechanism Analysis

In this subsection, we present the theoretical analysis, demonstrating that FIMI can achieve the desired properties.

Lemma 1. *FIMI is RTW feasible.*

Proof. FIMI obtains k interchangeable user set pairs: $(S_1, S'_1), (S_2, S'_2), ..., (S_k, S'_k)$, where $S'_C = U^k_{i=1} S_i, S''_C = U^k_{i=1} S'_i$. The winner set S is the set of winners in each interchangeable user set pair. Since both S'_C and S''_C are RTW feasible user sets, S must be RTW feasible user set too. □

To analyze the computational efficiency of FIMI, we need the following Lemma 2.

Lemma 2. *For any user $i \in S'_C$ (resp. S''_C), if there exists user $j \in S''_C$ (resp. S'_C) satisfying $s_j \leqslant e_i < e_j$, then the number of user j is at most* 2.

Proof. We assume there is user i in S'_C, and there are three users with ID 1, 2 and 3 respectively in S''_C satisfying

$$s_j \leqslant e_i < e_j, j \in \{1, 2, 3\}. \quad (3)$$

We firstly claim that s_1, s_2 and s_3 must be different from one another. This is because the user with a long UTW can replace the user with short one when they have the same start time, and S''_C is still feasible. Since S'_C and S''_C together is the optimal feasible solution with the minimum total cost, we obtain the claim above. Assuming $s_1 < s_2 < s_3$, we argue that user 2 and user 3 must not exist in S''_C simultaneously. There are $s_2 < e_1$ and $s_3 < e_1$ based on (3). In the case $e_2 \leqslant e_3$, user 2 can be removed, and S''_C is still feasible. In the case $e_2 > e_3$, user 3 can be removed, and S''_C is still feasible. As a conclusion, user 2 and user 3 must not exist in S''_C simultaneously. □

Lemma 3. *FIMI is computationally efficient.*

Proof. In the candidate selection phase, the time complexity of the selection function is dominated by finding the minimum cost flow (line 4 of Algorithm 2), which takes $O(n^2)$. In the payment function, we call the selection function for each winner (lines 2∼4 of Algorithm 3), thus the time for computing payment takes

$O(n^3)$ time. Hence the time complexity of candidate selection is bounded by $O(n^3)$. Then we analyze the time complexity of winner selection. Finding the intersection points of two paths (line 2 of Algorithm 4) takes $O(n^2)$ since there are at most $2n$ edges which cross both S'_C and S''_C based on Lemma 2, and for each of them, FIMI checks whether there exists another edge in order to generate an intersection point. Afterwards, choosing winners and calculating payments will take $O(n)$. Thus, the time complexity of the winner selection phase is bounded by $O(n^2)$. As a result, the time complexity of the whole FIMI is bounded by $O(n^3)$, where n is the number of user candidates. □

Remark. The time complexity of FIMI, $O(n^3)$, is very conservative since the number of winners is much smaller than n in practice. Note that the time complexity of VCG auction, which is a comparative mechanism illustrated in Subsection 4.5, is also $O(n^3)$.

Lemma 4. *FIMI is individually rational.*

Proof. We denote p_j^1 and p_j^2 as the payments calculated in the candidate selection phase and the winner selection phase to user j, respectively. Since p_j^1 is calculated based on VCG payment rule, which is known as an individually rational mechanism, we have $p_j^1 \geqslant b_j$. On the other hand, considering there are $k+1$ intersection points of paths A and A'. For any $i \in \{1, ..., k\}$, if S_i wins, i.e., $\sqrt{|S_i|} \times c(S_i) \leqslant \sqrt{|S'_i|} \times c(S'_i)$, the payment to user $j \in S_i$ in the winner selection phase is $p_j^2 = \sqrt{|S'_i|} \times \frac{c(S'_i)}{\sqrt{|S_i|}} - c(S_i) + b_j$. We obtain $p_j^2 \geqslant b_j$ and vice versa. As a conclusion, we have $p_j = \min\{p_j^1, p_j^2\} \geqslant b_j$. □

Lemma 5. *FIMI is truthful.*

Proof. Based on Theorem 1, it suffices to prove that the selection rule of FIMI is monotone and the payment p_j for each j is the critical value. The monotonicity of the selection rule is obvious as bidding a smaller value cannot push user j out of S.

We next show that p_j is the critical value for j in the sense that bidding higher p_j could prevent j from winning the auction. Note that $p_j = \min\{p_j^1, p_j^2\}$ based on FIMI. If user j bids $b_j \geqslant p_j^1$, he/she will lose in the candidate selection phase since p_j^1 is the critical value of VCG payment rule. Consider $p_j^2 = \sqrt{|S'_i|} \times \frac{c(S'_i)}{\sqrt{|S_i|}} - c(S_i) + b_j$ (if $j \in S_i$). If user j bids $b_j \geqslant p_j^2$, we have $\sqrt{|S_i|} \times c(S_i) > \sqrt{|S'_i|} \times c(S'_i)$, which means S'_i will win and user j will lose in the winner selection phase. The lemma still hold if $j \in S'_i$. □

Since we apply the frugal path auction to solve our PMUS problem, we have the following lemma, which has been proved in [17].

Lemma 6. *FIMI is* $2\sqrt{2}$*-frugal.*

Proof. This can be achieved according to the frugality theory[17]. □

The above lemmas together prove the following theorem.

Theorem 3. *FIMI is RTW feasible, computationally efficient, individually rational, truthful, and* $2\sqrt{2}$*-frugal.*

5 Extension to Multi-Coverage Requirement

In the previous section, we proposed a constant frugal incentive mechanism for time window coverage tasks, in which the RTW is covered at least once. Here, we extend it to a more practical scenario, in which the RTW needs to be covered more than once, i.e., the platform has multi-coverage requirement of the RTW, termed RTW multi-coverage. The extended FIMI achieves RTW multi-coverage, computational efficiency, individual rationality, truthfulness, and constant frugality.

5.1 Extended FIMI

Without loss of generality, we assume that the platform has multi-coverage requirement $\hat{r} \in \mathbb{Z}_+$ indicating the number of times the RTW to be covered at least. We can select $\hat{r}$ groups of RTW feasible user sets to meet this constraint.

With the constraint of multi-coverage requirement, there should be enough users to form at least $\hat{r}+1$ disjoint feasible winner sets. However in practice, we can reduce the coverage requirement if the condition cannot be satisfied. We can verify the number of disjoint feasible user sets by the selection function in the candidate selection phase without much cost.

The PMUS problem for multi-coverage requirement is intractable, and the mechanism proposed in Section 4 cannot be applied directly here because the process of finding interchangeable user subsets between $\hat{r}$ winner sets is complicated and impractical. However, we can apply the selection rule of weighted cost between $\hat{r}+1$ feasible user sets, achieving the constant frugality. Just like FIMI, the extended FIMI is a two-phase mechanism. Due to space limitation, we will focus on the differences from FIMI.

5.1.1 *Phase* 1: *Candidate Selection*

Since we aim to select $\hat{r}$ winner sets from $\hat{r}+1$ feasible user sets, the extended FIMI first selects $\hat{r}+1$

disjoint RTW feasible user sets $\hat{S}_1, \hat{S}_2, ..., \hat{S}_{\hat{r}+1}$ with the minimum $c(\hat{S}_i), i \in \{1, 2, ..., \hat{r}+1\}$. Let $c(\hat{S}_1) \leqslant c(\hat{S}_2) \leqslant ... \leqslant c(\hat{S}_{\hat{r}+1})$. This can be done by running minimum cost path algorithm $\hat{r}+1$ times in the interval graph defined in Subsection 4.3. The payment rule is the same with that in FIMI.

5.1.2 *Phase* 2: *Winner Selection*

To apply the weighted cost between multiple feasible user sets to achieve the constant frugality, we should set the weight for each $\hat{S}_i$. This can be obtained by solving:

$$\hat{r}\zeta_i\beta = \sum_{j \in \{1,2,...,\hat{r}+1\}, j \neq i} \zeta_j |\hat{S}_j|, \tag{4}$$

where the weight of $\hat{S}_i$ is $\frac{1}{\zeta_i}$, and β is an unknown constant.

To solve this equation system, we can subtract the i-th equation from the first one, and obtain (5). Substituting ζ_i into the first equation (when $i = 1$) of (4), canceling out ζ_1, we can obtain the $\hat{r}+1$ degree polynomial equation for β. Therefore, there must be a solution of (4).

$$\zeta_i = \frac{|\hat{S}_1| + \hat{r}\beta}{|\hat{S}_i| + \hat{r}\beta}\zeta_1, \quad 2 \leqslant i \leqslant \hat{r} + 1. \tag{5}$$

We now introduce the winner selection mechanism which is shown in Algorithm 5. The extended FIMI calculates the weight for each user set $\hat{S}_i$ and discards the user set $\hat{S}_\sigma$ with the largest weighted cost $\frac{c(\hat{S}_\sigma)}{\zeta_\sigma}$. The winner set is $\hat{S} = \cup_{i \neq \sigma}\hat{S}_i$. In the end, we set the value of $\hat{p}_i, i \in \hat{S}$, to the minimum value of payment calculated in the candidate selection phase and the winner selection phase, respectively.

Algorithm 5. Extended FIMI-Winner Selection

Input: set of users U, feasible user sets $\hat{S}_1, \hat{S}_2, ..., \hat{S}_{r+1}$, strategy bids $\boldsymbol{B}$, payment $\hat{\boldsymbol{p}}$

1 $\hat{S} \leftarrow \emptyset$;
2 **forall** $i \in \{1, 2, ..., \hat{r}+1\}$ **do**
3   Calculate ζ_i defined in (5);
4 $\sigma \leftarrow \arg\max_{i \in \{1,2,...,\hat{r}+1\}} \frac{c(\hat{S}_i)}{\zeta_i}$;
5 **forall** $i \in \{1, 2, ..., \hat{r}+1\}$ **and** $i \neq \sigma$ **do**
6    $\hat{S} \leftarrow \hat{S} \cup \hat{S}_i$;
7 **forall** $\hat{S}_i \subset \hat{S}$ **do**
8   **forall** $j \in \hat{S}_i$ **do**
9      $\hat{p}_j \leftarrow \min\{\hat{p}_j, \frac{c(\hat{S}_\sigma)}{\zeta_\sigma}\zeta_i\}$;
10 Let $\hat{p}_i \leftarrow 0$ for all $i \in U \backslash \hat{S}$;
11 **return** $(\hat{S}, \hat{\boldsymbol{p}})$;

5.2 Mechanism Analysis

For the extended FIMI, we have the following theorem.

Theorem 4. *The extended FIMI achieves RTW multi-coverage, computational efficiency, individual rationality, truthfulness, and* 2*-frugality.*

Proof. The RTW multi-coverage is obvious since we select $\hat{r}$ user sets from $\hat{r}+1$ RTW feasible user sets.

For computational efficiency, in the candidate selection phase, the running time of the selection function is $O((\hat{r}+1)n^2)$ since we run minimum cost path algorithm $\hat{r}+1$ times in order to find $\hat{r}+1$ feasible user sets. In the payment function, we call the selection function for each winner; thus computing payment takes $O((\hat{r}+1)n^3)$ time. Hence the running time of candidate selection is bounded by $O((\hat{r}+1)n^3)$. In the winner selection phase of the extended FIMI (Algorithm 5), the running time of selection rule (lines 1~8) is dominated by solving the equation system (lines 2~4), which will take $O((\hat{r}+1)^3)$ at most. The payment decision (lines 9~14) will take $O(n)$ since there are at most n users. Note that $\hat{r}+1 \leqslant n$ because the extended FIMI first selects $\hat{r}+1$ disjoint RTW feasible user sets from n users. As a result, the running time of the whole extended FIMI is bounded by $O((\hat{r}+1)n^3)$.

Next, we show that the extended FIMI is individually rational. We denote $\hat{p}_j^1$ and $\hat{p}_j^2$ as the payments calculated in the candidate selection phase and the winner selection phase of the extended FIMI respectively. We have $p_j^1 \geqslant b_j$ based on Lemma 4. In the winner selection phase, if $\hat{S}_i$ wins, we have $\frac{c(\hat{S}_\sigma)}{\zeta_\sigma} \geqslant \frac{c(\hat{S}_i)}{\zeta_i} \geqslant \frac{b_j}{\zeta_i}$ for any $1 \leqslant i \leqslant \hat{r}+1, j \in \hat{S}_i$. Thus, the payment to user j is $\hat{p}_j^2 = \frac{c(\hat{S}_\sigma)}{\zeta_\sigma}\zeta_i \geqslant \frac{c(\hat{S}_i)}{\zeta_i}\zeta_i \geqslant b_j$. Thus, we have $\hat{p}_j = \min\{\hat{p}_j^1, \hat{p}_j^2\} \geqslant b_j$.

For the truthfulness, the monotonicity of the selection rule is obvious as bidding a smaller value cannot push user j out of $\hat{S}$. We next show that $\hat{p}_j = \min\{\hat{p}_j^1, \hat{p}_j^2\}$ is the critical value for j. If user j bids $b_j \geqslant \hat{p}_j^1$, he/she will lose in the candidate selection phase since $\hat{p}_j^1$ is the critical value of VCG payment rule. Considering $p_j^2 = \frac{c(\hat{S}_\sigma)}{\zeta_\sigma}\zeta_i$ (if $j \in \hat{S}_i$), if user j bids $b_j \geqslant \hat{p}_j^2$, we have $\frac{b_j}{\zeta_i} \geqslant \frac{c(\hat{S}_i)}{\zeta_i} \geqslant \frac{c(\hat{S}_\sigma)}{\zeta_\sigma}$, which means $\hat{S}_i$ will lose.

The proof of the frugality of the extended FIMI is similar to that of Theorem 10 of [17]. □

Remark. Our mechanisms still work when the platform has heterogeneous tasks with different RTWs. We can apply FIMI or extended FIMI to each of the RTWs straightforwardly, and the five desirable properties still hold.

6 Performance Evaluation

We conduct thorough simulations to investigate the performance of FIMI. We first evaluate FIMI based on real-world experience data traces. Then the randomly generated user based simulations are conducted in order to reveal the impacts of the key parameters. We measure the number of winners, the social cost, the total payment, and the running time in each instance. The bid price is uniformly distributed in [1, 100] for our simulations. The mechanisms are run on a Windows machine with Intel Core i5-4210U CPU and 4 GB memory. All the results are averaged over 1 000 runs.

6.1 Evaluation Based on Real Traces

We use the real mobility traces of 370 taxi cabs that report their position every 15 seconds around the city of Rome during 2014-02-01~2014-03-02[38]. For our simulations, we use the traces at the time snapshot on 2014-02-01. We consider that the time window coverage tasks are launched in the specific geographical areas. We choose two places, Quirinal Palace (Quirinal) and the University of Arkansas Rome Center (UARC), as the centers of the specific geographical areas, which are the circular areas with a radius of 1 km. We assume that a smartphone is carried by the passenger or the driver of each taxi. For each geographical area, we fix the maximum RTW and measure the performance with different end time. The RTWs of Quirinal area and UARC area are [05:38:54, 08:58:53] and [20:20:22, 23:40:21], respectively, both with length 12 000 seconds. The bidders of each geographical area are taxis who are in this area during the RTW. We select the maximum length time interval in the RTW of each taxi as the UTW. The number of taxis involved at different end time is shown in Fig.8.

Since the start time is the same, the different end time imply different $|\mathcal{W}|$, which is an indication of the workload for the crowdsensing application. As can be seen from Fig.9, the number of winning taxis increases with the increase of $|\mathcal{W}|$ because the platform has to recruit more participants to accomplish the tasks in large RTWs. The winners of FIMI are fewer than those of VCG auction since FIMI prefers selecting fewer users. The VCG auction always achieves the minimum social cost; however, the total payment of FIMI is only 85.59% of that of VCG auction in Quirinal area and 84.51% in UARC area on average. We compute the benchmark of our PMUS problem for each end time based on Definition 1. As can be shown from Fig.9(c), the frugality ratios of FIMI on uniformly distributed true costs are 1.31 and 1.20 in Quirinal area and UARC area on average, compared with 1.53 and 1.43 of VCG auction on the same true costs, respectively. Moreover, the running time of FIMI increases when $|\mathcal{W}|$ goes up since the running time depends on the number of involved taxis. For the running time, there is little difference between two mechanisms since they have the same time complexity in theory. However, the running time of FIMI is bounded by 8.1 ms and 16.2 ms in Quirinal area and UARC area respectively when $|\mathcal{W}| = 12\,000$ s.

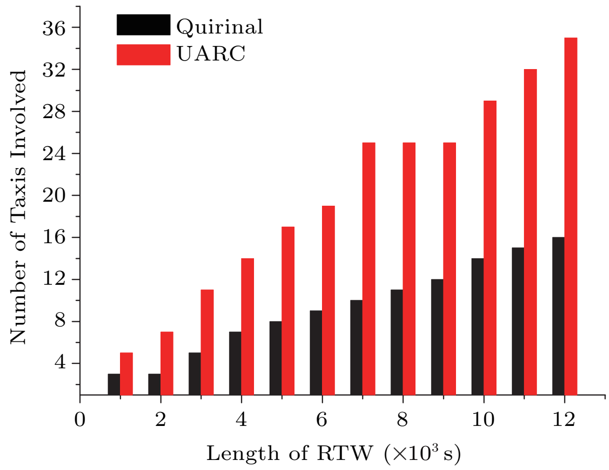


Fig.8. Taxis involved at different end time of RTWs.

For the extended FIMI, to meet the multi-coverage requirement, we use the traces at the different time snapshots in Quirinal Palace. The RTWs are [06:16:36, 08:47:40] for $\hat{r} = 2$, [06:19:27, 08:01:55] for $\hat{r} = 3$, and [06:29:35, 08:00:43] for $\hat{r} = 4$. Fig.10 shows the performance of extended FIMI with different multi-coverage requirements. We use the symbols of Extended FIMI($\hat{r}$), VCG($\hat{r}$) and Benchmark($\hat{r}$) to represent the extended FIMI, VCG and Benchmark with $\hat{r}$-coverage requirements, respectively. We observe that the larger $\hat{r}$, the larger the number of winners. This is because the platform has to recruit more participants to meet the coverage requirement. Accordingly, social cost, total payment, and running time also increase with $\hat{r}$. Social cost increases with the number of winners since bid price satisfies uniform distribution. In the extended FIMI, the platform has to pay each winning group of users; thus the total payment is closely

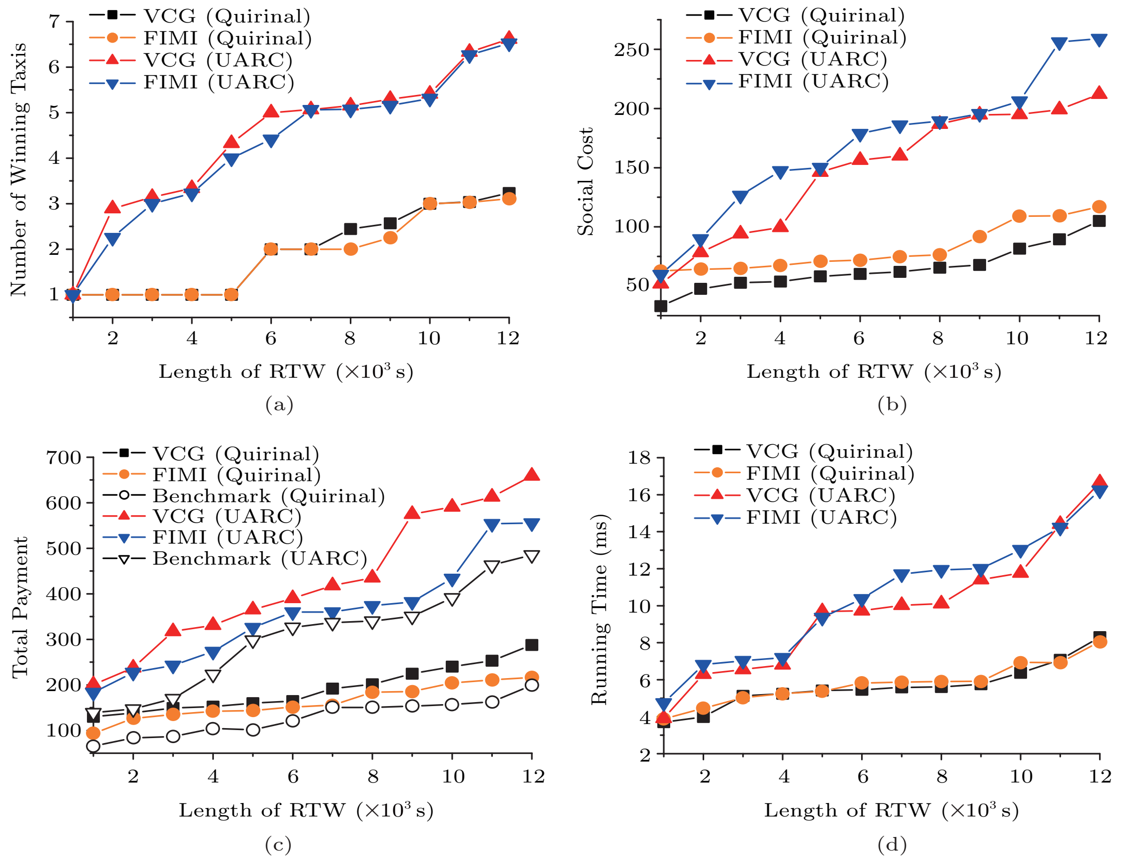


Fig.9. Performance of FIMI with different end time of RTWs. (a) Winning taxis. (b) Social cost. (c) Total payment. (d) Running time.

related to $\hat{r}$. However, the total payment of the extended FIMI is only 85.9% of that of VCG auction on average. The frugality ratios of FIMI on uniformly distributed true costs are 1.21 on average, compared with 1.41 of VCG auction on the same true costs. Moreover, running time increases when the value of $\hat{r}$ goes up since the time complexity of the extended FIMI is related to $\hat{r}$. However, the running time of the extended FIMI is bounded by 25.1 ms when $|\mathcal{W}| = 5\,000$ s and $\hat{r} = 4$.

6.2 Impacts of Key Parameters

There are three common key parameters: the number of users n, the length of RTW $|\mathcal{W}|$, and the upper limit ratio of UTW δ. For our simulations, the UTW length of each bid is uniformly distributed in the interval $[1, \delta|\mathcal{W}|]$. The UTWs are placed in the whole $\mathcal{W}$ with uniform distribution. We set $n = 1\,000$, $|\mathcal{W}| = 100, \delta = 0.2$ as the default values; however we will vary them for exploring the impacts of these parameters respectively. The impact of $|\mathcal{W}|$ has been investigated in Subsection 6.1. Thus we measure the impacts of other key parameters here.

6.2.1 *Impact of the Upper Limit Ratio of UTW*

The length of UTWs can depict the interest and suitability of users for participating in mobile crowdsensing. We set the UTW length of each bid in $[1, \delta|\mathcal{W}|]$ with uniform distribution, and then vary δ from 0.1 to 0.28. As can be shown in Fig.11, the number of winners and the social cost also decrease severely both in VCG auction and FIMI with increasing δ. This is because the platform can select fewer users to perform the tasks when each user can sense more data within $\mathcal{W}$. The number of winners in FIMI is 96.3% of that in VCG auction. The social cost of FIMI is about 10% higher than the optimal social cost. The total payment also decreases when the length of UTW goes up. However, the total payment in FIMI is lower than that of VCG auction in all cases, and is 90.68% of that in VCG auction on average. The frugality ratios of FIMI on uniformly distributed true costs are 1.08 on average, compared with 1.19 of VCG auction on the same true costs.

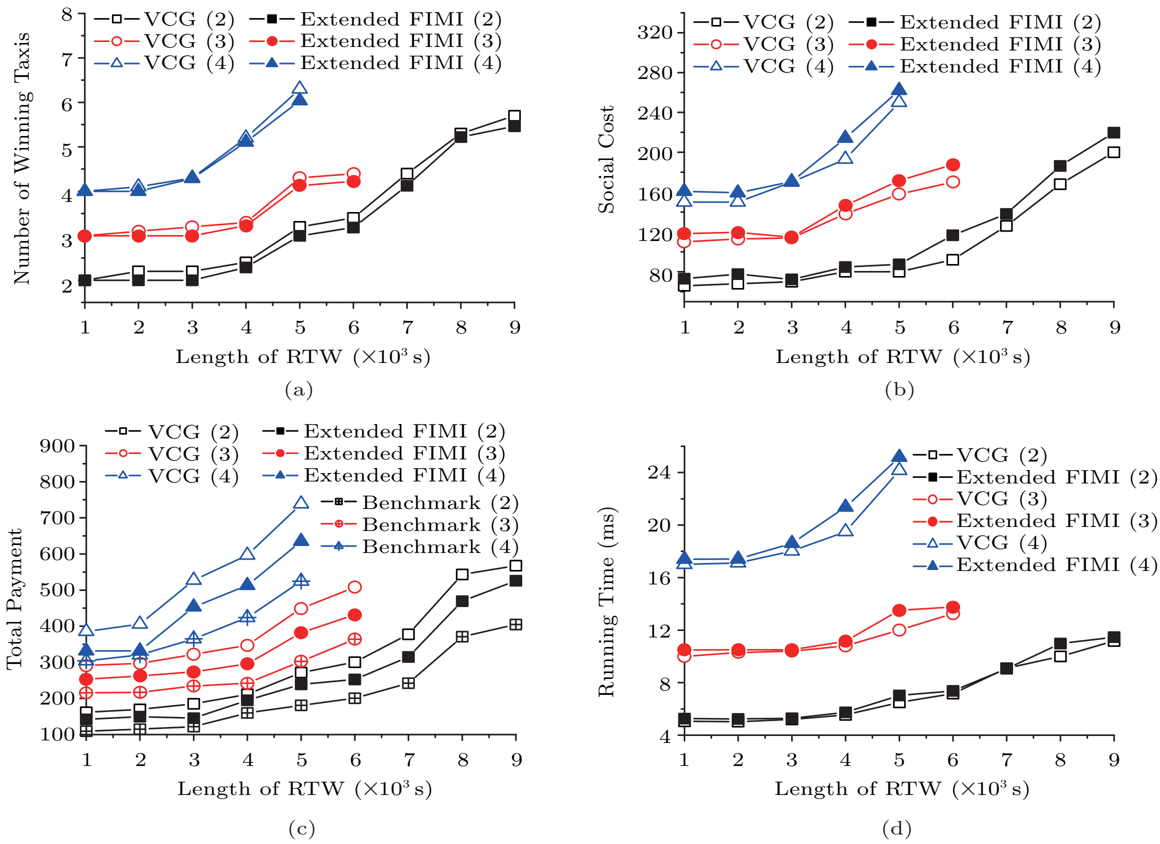


Fig.10. Performance of extended FIMI with different end time of RTWs. (a) Winning taxis. (b) Social cost. (c) Total payment. (d) Running time.

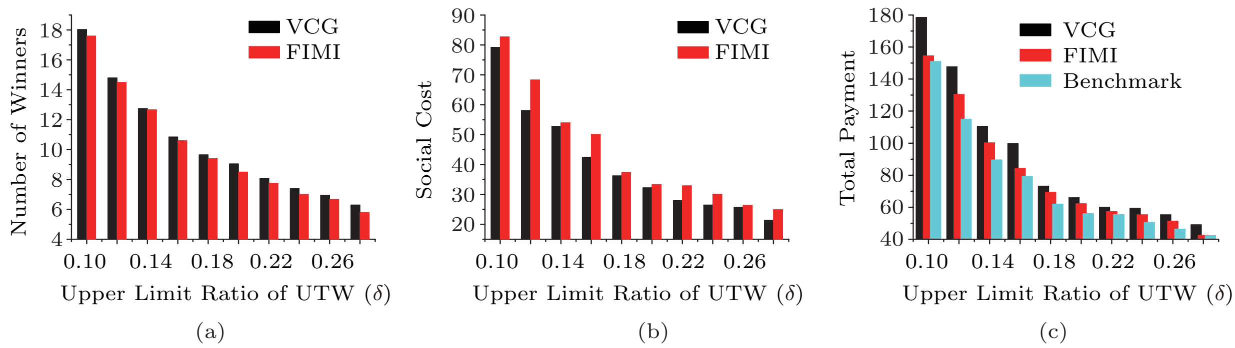


Fig.11. Impact of the upper limit ratio of user time window δ. (a) Winners. (b) Social cost. (c) Total payment.

6.2.2 *Impact of the Number of Users*

To investigate the scalability of designed mechanisms, we vary the number of users from 1 000 to 1 900. Fig.12 shows the impact of the number of users on the performance of FIMI. The number of winners in FIMI is less than that of VCG auction in all cases. The winner numbers of VCG auction and FIMI distribute from 8.47 to 9.05 and 8.07 to 8.50, respectively. Both of them do not change much when the number of users goes up. This is because that increasing the number of users cannot help to cover the RTW since the length of UTW of each bid is fixed. The social cost decreases with increasing the number of users since the platform can find more cheap users to perform the sensing tasks. The total payment also decreases when there are more participants. The total payment of FIMI is reduced by 11.99% compared with that of VCG auction. The frugality ratios of FIMI on uniformly distributed true costs are 1.11 on average, compared with 1.28 of VCG auction on the same true costs. Moreover, the running time

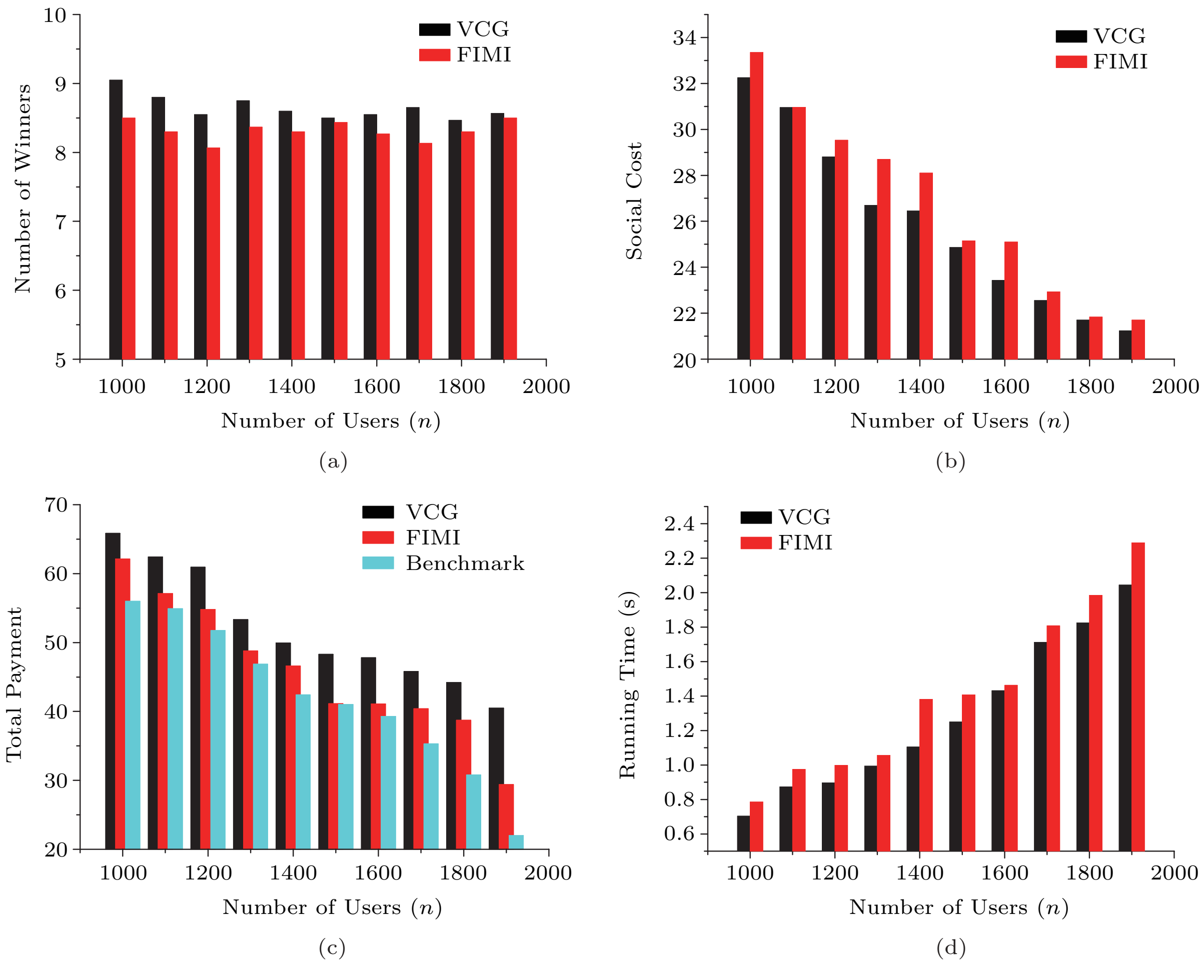


Fig.12. Impact of the number of users n. (a) Winners. (b) Social cost. (c) Total payment. (d) Running time.

increases with the increasing user scale. However, the designed mechanisms are computational efficient since the running time of FIMI is bounded by 2.29 s when the number of users reaches 1 900.

7 Conclusions

In this paper, we investigated the frugal truthful incentive mechanism for time window coverage in mobile crowdsensing. We presented a system model based on reverse auction and formulated the PMUS problem. We designed a constant frugal incentive mechanism, FIMI, which performs the auction in two phases: candidate selection and winner selection. Further, we extended FIMI to support RTW multi-coverage requirement. Through rigorous theoretical analyses, we demonstrated that the proposed mechanisms achieve RTW feasibility (or RTW multi-coverage), computation efficiency, individual rationality, truthfulness, and constant frugality. The results of extensive simulations showed that our mechanisms can reduce the total payment to 85% of that in VCG auction on average.

In the future work, we will consider the more complex scenarios. First, the time window coverage can be associated with specific locations. Second, the bids responded by the users can be extended to that including multiple time windows.

References

[1] Zheng Y D, Cheng R, Maniu S, Mo L Y. On optimality of jury selection in crowdsourcing. In *Proc. the 18th Int. Conf. Extending Database Technology*, March 2015, pp.193-204.

[2] Zheng Y D, Li G L, Li Y B, Shan C H, Cheng R. Truth inference in crowdsourcing: Is the problem solved? *Proc. the VLDB Endowment*, 2017, 10(5): 541-552.

[3] Zheng Y D, Li G L, Cheng R. DOCS: A domain-aware crowdsourcing system using knowledge bases. *Proc. the VLDB Endowment*, 2016, 10(4): 361-372.

[4] Li G L, Wang J N, Zheng Y D, Franklin M J. Crowdsourced data management: A survey. *IEEE Trans. Knowledge and Data Engineering*, 2016, 28(9): 2296-2319.

[5] Rana R K, Chou C T, Kanhere S S, Bulusu N, Hu W. Earphone: An end-to-end participatory urban noise mapping system. In *Proc. the 9th ACM/IEEE Int. Conf. Information Processing in Sensor Networks*, April 2010, pp.105-116.

[6] Hu H Q, Li G L, Bao Z F, Cui Y, Feng J H. Crowdsourcing-based real-time urban traffic speed estimation: From trends to speeds. In *Proc. the 32nd IEEE Int. Conf. Data Engineering*, May 2016, pp.883-894.

[7] Ye H B, Gu T, Tao X P, Lv J. Infrastructure-free floor localization through crowdsourcing. *Journal of Computer Science and Technology*, 2015, 30(6): 1249-1273.

[8] Zhou P F, Zheng Y Q, Li M. How long to wait?: Predicting bus arrival time with mobile phone based participatory sensing. In *Proc. the 10th ACM Int. Conf. Mobile Systems, Applications, and Services*, June 2012, pp.379-392.

[9] Jiang Z P, Xi W, Li X Y, Tang S Z, Zhao J Z, Han J S, Zhao K, Wang Z, Xiao B. Communicating is crowdsourcing: Wi-Fi indoor localization with CSI-based speed estimation. *Journal of Computer Science and Technology*, 2014, 29(4): 589-604.

[10] Yang D J, Xue G L, Fang X, Tang J. Crowd-sourcing to smartphones: Incentive mechanism design for mobile phone sensing. In *Proc. the 18th Annual Int. Conf. Mobile Computing and Networking*, August 2012, pp.173-184.

[11] Singer Y, Mittal M. Pricing mechanisms for crowdsourcing markets. In *Proc. the 22nd Int. Conf. World Wide Web*, May 2013, pp.1157-1166.

[12] Feng Z N, Zhu Y M, Zhang Q, Ni L M, Vasilakos A V. TRAC: Truthful auction for location-aware collaborative sensing in mobile crowdsourcing. In *Proc. IEEE INFOCOM*, April 2014, pp.1231-1239.

[13] Zhao D, Li X Y, Ma H D. How to crowdsource tasks truthfully without sacrificing utility: Online incentive mechanisms with budget constraint. In *Proc. IEEE INFOCOM*, April 2014, pp.1213-1221.

[14] Zhang X L, Yang Z, Zhou Z M, Cai H B, Chen L, Li X Y. Free market of crowdsourcing: Incentive mechanism design for mobile sensing. *IEEE Trans. Parallel and Distributed Systems*, 2014, 25(12): 3190-3200.

[15] Singer Y. Budget feasible mechanisms. In *Proc. the 51st Annual IEEE Symp. Foundations of Computer Science*, October 2010, pp.765-774.

[16] Archer A, Tardos É. Frugal path mechanisms. *ACM Trans. Algorithms*, 2007, 3(1): Article No. 3.

[17] Karlin A R, Kempe D. Beyond VCG: Frugality of truthful mechanisms. In *Proc. the 46th Annual IEEE Symp. Foundations of Computer Science*, October 2005, pp.615-624.

[18] Nisan N, Roughgarden T, Tardos E, Vazirani V V. Algorithmic Game Theory. Cambridge: Cambridge University Press 2007.

[19] Zheng Y D, Wang J N, Li G L, Cheng R, Feng J H. QASCA: A quality-aware task assignment system for crowdsourcing applications. In *Proc. the ACM SIGMOD Int. Conf. Management of Data*, May 2015, pp.1031-1046.

[20] Fan J, Li G L, Ooi B C, Tan K I, Feng J H. iCrowd: An adaptive crowdsourcing framework. In *Proc. the ACM SIGMOD Int. Conf. Management of Data*, May 2015, pp.1015-1030.

[21] Hu H Q, Zheng Y D, Bao Z F, Li G L, Feng J H, Cheng R. Crowdsourced POI labelling: Location-aware result inference and task assignment. In *Proc. the 32nd IEEE Int. Conf. Data Engineering*, May 2016, pp.61-72.

[22] Kazemi L, Shahabi C. GeoCrowd: Enabling query answering with spatial crowdsourcing. In *Proc. the 20th Int. Conf. Advances in Geographic Information Systems*, November 2012, pp.189-198.

[23] Tong Y X, She J Y, Ding B L, Wang L B, Chen L. Online mobile micro-task allocation in spatial crowdsourcing. In *Proc. the 32nd IEEE Int. Conf. Data Engineering*, May 2016, pp.49-60.

[24] Tong Y X, She J Y, Ding B L, Chen L, Wo T Y, Xu K. Online minimum matching in real-time spatial data: Experiments and analysis. *Proc. the VLDB Endowment*, 2016, 9(12): 1053-1064.

[25] Zhang X, Xue G L, Yu R Z, Yang D J, Tang J. Truthful incentive mechanisms for crowdsourcing. In *Proc. IEEE Conf. Computer Communications*, April 2015, pp.2830-2838.

[26] Xu J, Li H, Li Y X, Yang D J, Li T. Incentivizing the biased requesters: Truthful task assignment mechanisms in crowdsourcing. In *Proc. the 14th Annual IEEE Int. Conf. Sensing, Communication, and Networking*, June 2017.

[27] Koutsopoulos I. Optimal incentive-driven design of participatory sensing systems. In *Proc. IEEE INFOCOM*, April 2013, pp.1402-1410.

[28] Xu J, Xiang J X, Yang D J. Incentive mechanisms for time window dependent tasks in mobile crowdsensing. *IEEE Trans. Wireless Communications*, 2015, 14(11): 6353-6364.

[29] Xu J, Xiang J X, Li Y X. Incentivize maximum continuous time interval coverage under budget constraint in mobile crowd sensing. *Wireless Networks*, 2017, 23(5): 1549-1562.

[30] Talwar K. The price of truth: Frugality in truthful mechanisms. In *Proc. Annual Symp. Theoretical Aspects of Computer Science*, February 2003, pp.608-619.

[31] Kempe D, Salek M, Moore C. Frugal and truthful auctions for vertex covers, flows and cuts. In *Proc. the 51st Annual IEEE Symp. Foundations of Computer Science*, October 2010, pp.745-754.

[32] Elkind E, Goldberg L A, Goldberg P W. Frugality ratios and improved truthful mechanisms for vertex cover. In *Proc. the 8th ACM Conf. Electronic Commerce*, June 2007, pp.336-345.

[33] Chen N, Elkind E, Gravin N, Petrov F. Frugal mechanism design via spectral techniques. In *Proc. the 51st Annual IEEE Symp. Foundations of Computer Science*, October 2010, pp.755-764.

[34] Hsu W J, Dutta D, Helmy A. CSI: A paradigm for behaviororiented profile-cast services in mobile networks. *Ad Hoc Networks*, 2012, 10(8): 1586-1602.

[35] Myerson R B. Optimal auction design. *Mathematics of Operations Research* 1981, 6(1): 58-73.

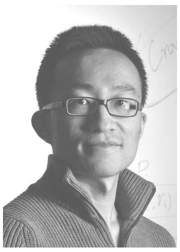
[36] Immorlica N, Karger D, Nikolova E, Sami R. First-price path auctions. In *Proc. the 6th ACM Conf. Electronic Commerce*, June 2005, pp.203-212.

[37] Edmonds J, Karp R M. Theoretical improvements in algorithmic efficiency for network flow problems. *Journal of the ACM*, 1972, 19(2): 248-264.

[38] Amici R, Bonola M, Bracciale L, Rabuffi A, Loreti P, Bianchi G. Performance assessment of an epidemic protocol in VANET using real traces. *Procedia Computer Science*, 2014, 40: 92-99.

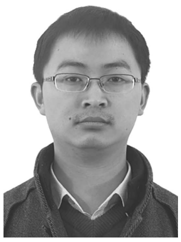
Jia Xu received his M.S. degree in computer science from Yangzhou University, Yangzhou, in 2006, and his Ph.D. degree in computer science from Nanjing University of Science and Technology, Nanjing, in 2010. He is currently a professor at Nanjing University of Posts and Telecommunications, Nanjing. He was a visiting scholar in the Department of Electrical Engineering and Computer Science, Colorado School of Mines, Golden, from November 2014 to May 2015. His main research interests include crowdsourcing, opportunistic networks, and wireless sensor networks. He is a member of CCF, ACM and IEEE.

Jian-Ren Fu received his Bachelor's degree in computer science from Shaoyang University, Shaoyang, in 2014. He is currently working toward his Master's degree in the School of Computer Science at Nanjing University of Posts and Telecommunications, Nanjing. His research interests are mainly in the area of the mobile crowdsensing, wireless sensor networks, and wireless ad-hoc networks.



De-Jun Yang received his B.S. degree in computer science from Peking University, Beijing, in 2007, and his Ph.D. degree in computer science from Arizona State University, Tempe, in 2013. He is currently the Ben L. Fryrear Assistant Professor in the Department of Computer Science at Colorado School of Mines, Golden. His main research interests include economic and optimization approaches to networks, crowdsourcing, smart grid, big data, and cloud computing. He has received Best Paper Awards at IEEE MASS 2011 and IEEE ICC 2011 and 2012, and was a Best Paper Award Runner-up at IEEE ICNP 2010. He is a member of IEEE.

Li-Jie Xu received his Ph.D. degree in the Department of Computer Science and Technology from Nanjing University, Nanjing, in 2014. He was a research assistant in the Department of Computing at the Hong Kong Polytechnic University, Hong Kong, from 2011 to 2012. He is currently an assistant professor in the School of Computer Science at Nanjing University of Posts and Telecommunications, Nanjing. His research interests are mainly in the areas of wireless sensor networks, ad-hoc networks, mobile and distributed computing, and graph theory algorithms.



Lei Wang received his B.S. degree from Anhui University, Hefei, in 2008, and his Ph.D. degree in computer science from Nanjing University of Science and Technology, Nanjing, in 2014. He was a Master-Doctor combined program graduate student of Nanjing University of Science and Technology in 2008. He was a visiting student in Michigan State University, East Lansing, during 2013~2014. Currently, he is an assistant professor in the School of Computer Science at Nanjing University of Posts and Telecommunications, Nanjing. His research interests are network coding, wireless sensor network, multipath routing, and distributed storage system.

Tao Li received his Ph.D. degree in computer science in 2004 from the University of Rochester, Rochester. He is currently a professor in the School of Computer Science at Nanjing University of Posts and Telecommunications, Nanjing. His research interests are in data mining, computing system management, and information retrieval. He is a recipient of USA NSF CAREER Award and multiple IBM Faculty Research Awards. He is a member of IEEE.